THE SPECIALISTS' SERIES

ELECTRICAL INSTRUMENTS

IN

THEORY AND PRACTICE

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WITH 164 ILLUSTRATIONS

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PREFACE

During the course of many years lecturing and working with instruments we have found the necessity for some work dealing with the theory beyond merely the first terms. It is hoped this book will meet with the approval of students in the higher classes, and of engineers.

In many cases, and especially in the section on electric supply meters, the results of numerous experiments are compared with theory. The theory of the induction meter is extremely complex, and the explanations given are sometimes quite erroneous. We hope the views we set forth will prove useful.

The instruments which have been selected are merely typical of those in use, and we propose completing the work by dealing with oscillographs, ondographs, and many alternating current instruments both theoretically and experimentally in another volume.

The figures have either been copied from original papers referred to in the text, from catalogues of the

makers, or drawn directly from the instruments themselves.

For purely descriptive work the student must refer to the many excellent treatises already on the market, and for further information as to theory where space is too restricted, to the numerous references given throughout the text.

W. H. F. M.
U. A. O.

London, 1915.

CONTENTS

ERRATA

Page 13, paragraph on p. 41, "We now proceed," etc., should precede "Methods of Measurement," p. 13.

,, 76, Formula at top should read :

$$\frac{(\mathrm{BAN})^2 \sin\theta \,.\, \cos\theta \,.\, \pi n \sin\phi}{10^9 \sqrt{\mathrm{R}^2 + (2\pi n \mathrm{L})^2}},$$

,, 108, delete decimal point in line opposite Resistance

,, 112, delete cyclic currents in diagram.

,, 117, nine lines from foot of page, read "*bl*" for "*ρl*."

,, 221, line 12, read "moment" instead of "movement."

,, 239, read "Weston Thomson Commutator Meter" in diagram.

,, 243, line 11, read "time taken to come to rest."

,, 315, line 5, read "coercive force," and "on" next line.

,, 365, in index, read "Schwendler" for "Swendlem."

ELECTRICAL INSTRUMENTS

CHAPTER I

HISTORICAL SUMMARY

REGARDING the testing instruments dealt with in this book it may be said that their origin is of comparatively recent date.

Cavendish, one of the earliest electrical experimenters, obtained, by means of what would nowadays be regarded as extremely rough apparatus, definite numerical values for various quantities, such as capacities of spheres and discs. His method also of proving the inverse square law as regards electrostatic forces is at present by far the best.

In fact it was in the endeavour to test the various laws experimentally that instruments of precision began to be evolved.

Consequently we had "unit phials" as units of capacity, the torsion balance, the attracted disc electrometer of Snow Harris, and the magnetometer of Gauss.

Much was done previous to the advent of any rational system of units, and it is sometimes forgotten that "Ohm's Law" was not originally expressed in the form—

$$\text{Ohms} = \frac{\text{Volts}}{\text{Amperes}}$$

(*vide Die Galvanomische kette mathematische Bearbeiten,* 1827), and that when it was discovered, these units were non-existent.

The advent of the electric telegraph and the commercial importance attached to electric signalling undoubtedly gave a great impetus to invention, and when the instruments were required they were frequently evolved.

The laws of attraction or repulsion of electric currents had already been worked out by Ampère, and it remained for those engaged in cable laying, or telegraphic work, to obtain a system of units and to have suitable testing apparatus.

Currents could be measured by means of tangent galvanometers—Joule invented an improved type about 1843 with fibre suspension and small needles—or by deposition of copper or silver with a voltameter, but neither of these methods is particularly suitable for practical work, since the former is "insensitive," and the latter requires time and care. The Tangent Galvanometer, however, in Post Office form, was long used for testing land lines.

For cable testing or signalling the mirror galvanometer was devised by Lord Kelvin, and also, for work on board ship, a "marine" galvanometer was required. In his marine galvanometer Lord Kelvin introduced a permanent magnet to replace the field due to the earth, and the suspension of the galvanometer mirror and needles was secured top and bottom. This enabled the deflection to be read while the ship was in motion.

Again Weber had invented the dynamometer to measure strengths of current. It consisted of a small coil suspended bifilarly, and at right angles to the plane

of a larger coil. When the current passed through the two coils in series the plane of the large coil required to be turned through some angle θ, the small coil being brought into the magnetic meridian. In this case the only couple acting on the small coil was due to its suspension, supposed unifilar, and it is easily seen that the current is given by

$$C = \sqrt{a \cdot \frac{\theta}{\cos \theta}},$$

where a is a constant. The student may as well note the form of the expression, which does not differ greatly from the formulæ for certain instruments to be discussed later.

A strong permanent magnetic field was also used in Lord Kelvin's syphon recorder, and a movement comprising moving coil working an inking pen. By this means the effect of the earth's field was eliminated altogether, and since the magnetic field was invariable the deflections under suitable circumstances may depend only on the strength of current.

This apparently led to the use of the so-called D'Arsonval type of galvanometers, so common at the present time.

Daniell's primary battery in various forms gave a convenient standard of electromotive force, until it was replaced by Clark's or Weston's standards used nowadays as being more reliable.

As will be seen, considerations of "sensitiveness" led to the development of galvanometers having the coils close to the moving needle, and questions as to the method of winding coils to produce the best result led to improved mirror and tangent galvanometers.

Amongst the earliest instruments for measuring currents or E.M.F.'s were, of course, Siemens's dynamometer and Lord Kelvin's graded galvanometers.

The former were instruments consisting of a fixed and a movable coil, the same current passing through them both in series. The control was due to a spring attached to a torsion head which could be turned, as in Coulomb's torsion balance. From this it followed that approximately,

$$C = k\sqrt{\theta},$$

where θ is the angle of torsion and k is a constant.

Again the graded galvanometers were merely tangent galvanometers wound with wire so "graded" that the cross-section of the wire was approximately proportional to the diameter of the coil at the point. The needle consisted of a small magnet with a pointer at right angles, and the scale was a tangent one. In using it, it had to be set in the magnetic meridian (unless controlled by a magnet), and the box and needle placed at different distances from the centre of the coil and on its axis.

Both the above instruments were affected by external magnetism, or the neighbourhood of a dynamo.

The dynamometer wattmeter was an obvious instrument after the invention of the dynamometer ammeter, and it possessed, of course, the advantage of a uniform scale.

At the same time as these instruments were being devised, Lord Kelvin had invented a series of electrometers—Portable, Quadrant, and Absolute—depending for their action on the attraction between electrified plates of metal or movements of electrified plates.

These instruments are described in *Electrostatics and*

Magnetism, p. 261, "Report on Electrometers and Electrostatic Measurements," *B.A. Report*, 1867.

It is almost needless to refer to the importance of the quadrant electrometer since in the improved form, which is little different from Lord Kelvin's, it is largely used as Dolezalek's pattern at the present time. The quadrant electrometer also led directly to the multicellular voltmeter of Kelvin and others.

Again, the absolute electrometer with its attracting discs improved by means of the famous "guard ring," not only gave us a means of avoiding errors in making measurements where lines of force become distorted towards the edges of a plate, but have led to the design of high tension voltmeters through a better understanding of the laws of electrostatic forces. The guard-ring device also has been used in the verification of the

$$\frac{B^2}{8\pi}$$

formula for magnetic traction effort (*vide* Dr. Taylor Jones, *Phil. Mag.* vol. 41, p. 153).

Together with these instruments electroscopes and galvanoscopes existed, and it is interesting to remember that the pith ball, or diverging straws, were used by Beccaria, Volta, and others in the seventeenth century. The gold-leaf electroscope was devised by Bennet, who also invented the famous "Doubler," and this gold-leaf electroscope in a much improved form by C. T. R. Wilson is used now as an electrometer for many purposes, particularly in connection with radio-activity measurements.

The Atlantic cable enterprise led to a critical examination of the way in which signals are transmitted by wires, and Lord Kelvin's papers on "Transient Electric

Currents" (*Math. and Phys. Papers*, vol. i. p. 540), and his "Theory of the Electric Telegraph" (*Math. and Phys. Papers*, vol. ii. p. 61), did a tremendous amount towards "clearing the air" of erroneous notions and showing the importance of capacity and self-induction in practice.

All the above work led again to the necessity for "dead beat" or "damped" instruments, and in Kelvin's mirror galvanometer this was attained more or less satisfactorily by enclosing the needle and mirror inside a small space so that the needle acted as a plunger in a dashpot. These instruments, except for certain purposes of illustration, have now passed away, and are replaced by more satisfactory types. To a man who was a student twenty-five years ago, he can truly say with Charles Lamb, on looking round a modern laboratory: "Gone, gone are all the old familiar faces."

At first, instruments of switchboard type for measuring volts and amperes were mostly of the moving iron plunger type. Lord Kelvin used a solenoid, "sucking" a "saturated" iron plunger into it, and many other types of instruments with soft iron movements were in vogue in 1890. At that time 150 kilowatts was considered a large steam set, and papers were read proving the uselessness of adopting large "units" of 10,000 kilowatts, or so. Consequently as the demand for light and power gradually grew, engineers were dissatisfied with the older types of iron-cored instruments, not only because they were not "dead beat" and subject to errors, but because they had a poor scale unevenly divided.

Hence the D'Arsonval galvanometer with a spring control was adopted and simply shunted with a suitable

shunt to measure large currents, or used with a series resistance to measure volts.

Owing to the invariable magnetic fields permanent magnet-moving coil instruments are no use for measuring alternating currents, or voltages.

Now in such cases as an instrument such as Siemens's dynamometer wattmeter, Kelvin's balance (first patent, No. 2028, April 21, 1883), or Kelvin's multicellular voltmeter where the force at any instant between the movable portions depended on the square of some harmonically varying quantity, it is easily seen that they measure

$$C_m = \sqrt{\frac{1}{T}\int_0^T c^2 dt}\ ;$$

where c is the instantaneous value of the quantity, T is the periodic time. The above expression is put into words by saying that such instruments measure "the square root of the time average of the sum of the squares" of the quantity in question.

Instruments again like Cardew's voltmeter were used freely about 1890 to measure on continuous or alternating circuits. The heating of the wire composing it depended on the square of the varying quantity, and so it satisfied the above requirements. Many types of hot wire instruments are now used.

An important principle which has been greatly utilised in galvanometer design is that of periodic time.

Roughly speaking we may say

$$I\frac{d^2\theta}{dt^2} = F_r$$

where F_r is the turning moment, and I the moment of inertia, the angular acceleration being

$$\frac{d^2\theta}{dt^2}.$$

This leads to the solution that

$$t = 2\pi\sqrt{\frac{I}{F_r}}.$$

Hence by making I very small and F_r very great the periodic time of any moving body can be reduced to a small quantity.

By taking advantage of this fact Blondel in France and Duddell in England invented the oscillograph, and by using oil or electromagnetic damping have rendered it possible to follow fluctuations of current or voltage down to 2000 cycles per second.

Previous to the use of oscillographs, Joubert's revolving contact maker was the only way in which the variations of such quantities as E.M.F. or alternating current could be studied. These instruments will be discussed in due course in the second volume.

In 1890 Lord Kelvin brought out a meter, an engine-room voltmeter and his multicellular voltmeter. His electrostatic voltmeter idiostatically arranged to measure from 400 to 10,000 volts was exhibited on April 20, 1887, and described in the *Proceedings of the Glasgow Philosophical Society*, pp. 249-256.

Several new types of electric balances were also described by him and descriptions of these will be found in the *Brit. Assoc. Reports*, 1887, the composite balance in 1888, *Glas. Phil. Soc. Trans.* vol. xix.

An interesting determination of the much discussed quantity "v" was made by Lord Kelvin, Professor W. F. Ayrton, and Professor J. Perry, *B.A. Report*, 1888, by comparing the readings of an "absolute" electrometer with the readings of a centi-ampere balance, the current passing through a known resistance. In this way a value of "v" of between 292×10^8 and 287×10^8 was obtained.

A Siemens dynamometer wattmeter and current meter was exhibited to the Glasgow Philosophical Society, January 9, 1884, together with Kelvin's potential galvanometer with astatic needles and methods of varying the sensibility by magnetic control, or by series resistance. At this meeting Lord Kelvin also exhibited his "mho meter" or conductivity meter, consisting of an astatic galvanometer with the steel controlling magnets replaced by fine wire coils of German silver perpendicular to the plane of the proper galvanometer coils.

His gravity control instruments were exhibited at B.A. meetings, 1885.

The use of instruments worked by coils which could be affected by external fields led to examination of the shielding effect of iron in this case, or tinfoil or wire cage in the case of electrostatic instruments. This had previously arisen in the case of ships' compasses. It might be noted, however, that in the magnetic field enclosed by a belt of iron 1 foot thick, 5 feet high, and 10 feet internal diameter, with roof and floor of comparatively thin iron, the average horizontal component of the earth's magnetism is only $\frac{1}{5}$th of the normal (*Math. and Phys. Papers*, vol. v. p. 523).

Electrostatic screening is considered in *Math. and Phys. Papers*, vol. v. p. 505. In this case it may be

necessary to use two screens if these are of the bird-cage type, and where there is a perforation, the force inside is not uniform within two or three diameters of the perforation. With ordinary wire netting the internal field is only $\frac{9}{10000}$ths approximately of the external. The use of double screens is interesting, as these results can be compared by the student with heat screening, as referred to by Fourier in his *Analytical Theory of Heat*, chap. ii., "Heating of Closed Spaces." Kelvin's quadrant electrometer was screened by strips of tinfoil forming coating of Leyden jar, the Dolezalek is usually placed inside a metallic box, and this box connected to earth through a water pipe. However, this will be dealt with in the proper place.

Again, air damping, or damping by a viscous fluid like oil, has given place to electromagnetic damping, and most instruments satisfy the requirement of being practically "dead beat."

Supply meters are all mostly of comparatively recent origin. The earliest was probably Edison's zinc deposition meter. The types at the present time are much improved in detail, but the original meters practically embodied the same principles. The great use of continuous current in this country for lighting has led largely to the adoption of coulomb meters, whether these are of the rotating mercury or motor type. Electrolytic meters are of necessity all coulomb meters.

After the reading of Hopkinson's now classical paper on "Dynamo-Electric Machinery" (*Phil. Trans. Roy. Soc.*, May 6, 1886, and February 15, 1892), in which great attention was directed to the magnetic properties of the iron composing the field magnets, and the line integral necessary to determine the windings, the importance

of magnetic testing was rendered more apparent than previously. Possibly, in consequence of this, practical methods of testing iron being required, instruments for this purpose were devised, and traction permeameters in 1888 and hysteresis testers about 1895 were used.

Ballistic galvanometers for measuring quantities of electricity were used by Faraday, and the only change is that nowadays they are of D'Arsonval type, or of the "flux meter" variety, which latter is independent of the rate of discharge.

Instruments for testing very high magnetic inductions of the order 30 to 40 thousand C.G.S. lines so far are not forthcoming, although for measuring inductions in the teeth of dynamo-armatures they would be most useful to designers.

Again, as regards instruments for measuring candle powers of lamps in terms of some standard, it seems strange to read nowadays that in 1881, the "most accurate and trustworthy" method then at hand was to do this by means of comparing the shadows of a pencil illumined by the two sources (*vide British Assoc. Report*, pp. 559-561, paper by Sir W. Thomson and J. T. Bottomley, "On the Illuminating Powers of Incandescent Vacuum Lamps with Measured Potentials and Measured Currents"). Since that time several different types of photometers have been invented, notably the Lummer-Brodhun screen and the various forms of Flicker photometers.

The rise of magnetic space telegraphy or wireless telegraphy has possibly tended towards giving an impulse to the invention of various kinds of instruments. Fedderson in 1859 had proved by photographing spark discharges (*Glasgow Phil. Soc. Proceedings*,

vol. iv. pp. 266-267; Lord Kelvin in "Photographed Images of Electric Sparks") that these were sometimes of an oscillating nature in accordance with Lord Kelvin's mathematical theory. Kelvin at once pointed out on reading this paper on the photographed sparks, that if the rate of rotation of the mirror and the distance from it of the plate receiving the impression were known, the "oscillation period" of the circuit could be determined. A considerable gap then ensued, and the subject of wireless telegraphy began to attract attention about 1894, and after that date "cymometers," or wave measurers, based on the oscillation equation $t=2\pi\sqrt{LK}$ were invented. The results have been compared by Campbell using a method not differing greatly from Fedderson's, sparks being photographed by a falling plate and the periodic time calculated (*Proceedings Phys. Soc.*, 1910).

Since it became important to ascertain the law according to which the strength of signals varied with distance, thermo-detectors were devised. The "bolometer" previously used by Langley was again utilised by Lieutenant C. Tissot, "*Étude de la Résonance des systèmes d'Antennae dans la télégraphie sans fils*," and values of the current at different distances measured.

The heating effort due to a train of waves was also measured by means of thermo-ammeters, where the heat generated in a resistance by the oscillatory current raised the temperature of a thermo-junction and caused a needle to give a deflection. Various forms of these instruments have been put on the market. At the same time the advent of wireless telegraphy in its various forms has encouraged the development of high frequency testing and resonance methods.

While in ordinary methods of testing resistance the current is assumed to be steady and obeying "Ohm's Law," in wireless work the current, being rapidly oscillatory, is mainly carried by the outside "skin" of the conductor. Formulæ or instruments based on the assumption of constant resistance or self-induction have, therefore, to be used with appropriate caution in such cases.

While telegraphy has been a most fruitful source so far as the invention of instruments is concerned, lighting and power taking second place, it is strange to reflect that so far telephony has added no new testing instruments of importance, and neither has electro-metallurgy.

Since resistance of a material to steady currents varies with its temperature, this principle has long been utilised to make electric resistance thermometers and pyrometers (see letter to Tyndall by C. W. Siemens, December 1860). More recently electric resistance thermometers have been greatly improved by Professor Callander. Electric pyrometers are also in common use in steel works to measure or record the temperature of furnaces.

It may be noted that where a delicate test of temperature is required, such instruments or an application of thermo-junctions are now almost a *sine qua non.*

Methods of Measurement

The tangent galvanometer improved by Joule or Helmholtz gives a simple means of measuring a current in absolute electromagnetic units.

Generally speaking, neglecting the torsion, or friction on the needle, we may write

$$C = \frac{Hr}{2\pi n} \tan \theta,$$

where r is the radius, n the number of turns, "H" the horizontal component of the earth's magnetic intensity, and θ the angle of deflection.

To a practical man interested in instruments this method certainly looks attractive, but apart from the difficulty in measuring "H" accurately, which is considerable, the fact remains that a tangent galvanometer is not a very sensitive instrument.

Besides the difficulty in measuring "H," it must not be forgotten that in this quantity there is a diurnal variation. Consequently "H" cannot be determined accurately to more than about three significant figures, or at least in conducting an experiment the fourth figure varies considerably. Consequently, this method is not used where the highest accuracy is required.

Currents are often measured by the electro-deposition of copper or silver, and in order to measure them absolutely the coefficient of electro-chemical decomposition must be determined by some absolute method.

Lord Rayleigh and Mrs. Sidgwick used a "current weigher." Since ampere balances are important instruments, we shall discuss this method briefly.

The apparatus consisted of fixed and movable coils arranged as shown in Fig. 1.

It will be noticed that the movable coil attached to the balance arm was placed in the central position and

between the two fixed coils. This was done to allow the maximum force to act upon it.

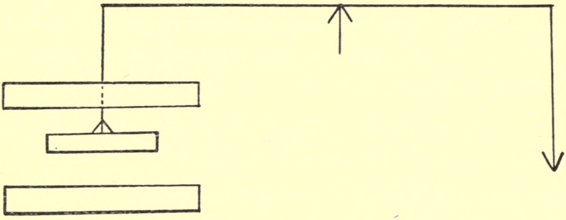

FIG. 1.—The Rayleigh balance. Fixed and movable coils.

Now if n_1, n_2 denote the number of lines of wire on the coils, M their mutual induction coefficient, and C the current strength, as is shown in treatises on electricity and magnetism, then the energy of the system is

$$T = n_1 n_2 C^2 M.$$

Differentiating with respect to x, the axial distance, we obtain for the force,

$$F = n_1 n_2 C^2 \frac{\partial M}{\partial x}.$$

Hence if F is balanced by means of a weight mg, we have

$$C = \sqrt{\frac{mg}{n_1 n_2 \frac{\partial M}{\partial x}}}$$

or
$$C = k\sqrt{m},$$

where k is a constant quantity.

It must be noticed that $\frac{\partial M}{\partial x}$ is a function merely of

distance apart and radii of coils, and it can be evaluated. The ratio of the radii of the coils was determined electrically by measuring their magnetic effect at a point at their centre, and corrections were made for the buoyancy in air by the moving coil. The final expression adopted was

$$C = 0{\cdot}037048\sqrt{m}.$$

We see, therefore, that if the current be kept constant for a definite time flowing through a silver solution, then if Faraday's laws are obeyed we have

$$W = C.\epsilon.t,$$

where W is the weight of metal deposited, C is the current, ϵ the electro-chemical equivalent, and t the time. Consequently we have

$$\frac{W}{\epsilon t} = 0{\cdot}037048\sqrt{m},$$

or

$$\epsilon = \frac{W}{0{\cdot}037048\sqrt{mt}},$$

so that the electro-chemical equivalent is determined absolutely against a mass m.

In carrying out the experiment, of course, many precautions had to be taken and the current was reversed in direction periodically, so that the attraction of the coils were sometimes acting for certain periods with gravity and sometimes against it. We see, therefore, that current can be measured absolutely by balancing its attractive forces against a mass acted upon by gravity.

Lord Kelvin also constructed balances depending for their action on the attraction between circuits carrying currents being balanced by some mass. As a general rule these instruments are not used as absolute ones, and they are generally calibrated by copper deposition.

It may as well be noticed that Kohlrausch measured "H" and "C" by means of passing current through two coils so arranged that one of them is acted upon by the earth's field, and at the same time passing the current through a galvanometer. Then two equations are obtained and both "H" and "C" can be determined (*Phil. Mag.* vol. xxxix., 1870).

Electromotive Force.—By means of the current weigher just discussed, Lord Rayleigh and Mrs. Sidgwick also determined the absolute electromotive force of Clark's standard cell by passing a known absolute current through a known absolute resistance then assuming Ohm's Law. (For a discussion on Ohm's Law see letters in *Electrician*, December 1913 *et seq.*)

$$E = CR.$$

The current weigher being balanced by weights from time to time and curve plotted weights vertically and time as abscissa, the exact current flowing when the electromotive force was balanced could be ascertained from it. A more direct method and a method more interesting instrumentally was that of Lord Kelvin with his absolute electrometer. This instrument consists of a fixed plate attracting a movable one. As shown in Fig. 2, the instrument consists of a Leyden jar forming the case of the instrument, a gauge, a replenisher. The air inside is kept dry by means of

pumice-stone soaked in sulphuric acid, placed in lead vessels at the bottom of the case.

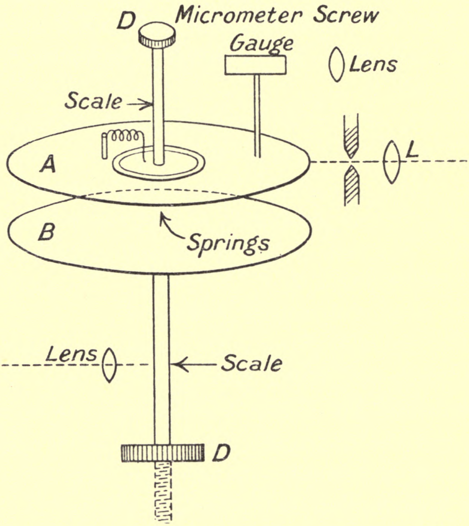

FIG. 2.—Kelvin absolute electrometer. Supporting springs for movable plate omitted.

The movable disc hangs in the circular aperture on the plate A, and the plate is supported by attachments to the jar. A gauge G is connected to this plate and the potential kept constant by means of the " replenisher," a small type of influence machine. By means of the disc turning screw at the top D, the movable plate can be raised or lowered against the action of a spring. In using the instrument the method of procedure was first to calibrate the spring attached to the movable plate by means of a dead weight—this sends index below the " sighted position " (see lens L) ; it is then raised to the

sighted position again and the distance noted by means of the micrometer screw D. The difference measures the distance through which an attraction equal to the weight would displace the plate. Six-tenths of a gramme displaced the plates by a distance corresponding to two complete turns of the head D.

The lower plate being then charged, the body whose potential is required is brought into contact with the movable plate by means of a wire and the disc D turned till the plate is in the sighted position. The plate can then be connected to earth and the distance again observed. The difference of the readings will then correspond to the difference of potential between the body and earth. Two observers were required, one to read the gauge and keep potential constant, the other to take readings.

The theory of the instruments is briefly as follows :—

To a very close approximation, if two plates are close together, the resultant electric force $\frac{\partial V}{\partial x}$ is given by $\frac{V}{D}$ where V is the electrostatic pressure or voltage and D the distance between them. And if ρ is the electrical density in the plate then

$$\rho = \frac{V}{4\pi D}.$$

The repelling force per unit of area is $2\pi\rho^2$, and therefore we have

$$F = \frac{V^2}{8\pi D^2},$$

consequently

$$V = D\sqrt{\frac{8\pi F}{A}}.$$

Since there is some difficulty in measuring the distance between the plates accurately, it is more convenient to measure their difference of distance with two different voltages to give the same force F.

In this way we find at once

$$V_1 - V_2 = (D_1 - D_2)\sqrt{\frac{8\pi F}{A}}.$$

Lord Kelvin found (April 12, 1860) that one thousand Daniell's cells with poles connected to two plates 1 *m/mm* apart and each a square diameter in area, produce an electrical attraction equal to the weight of 5·7 grammes (*vide Electrostatics and Magnetism*, p. 246).

Again it may be noted that Lord Kelvin had already calculated the E.M.F. of the Daniell cell from the expression

$$E = eJH,$$

where e is the electro-chemical equivalent, J is Joule's equivalent, and H the heat developed in chemical combination, and he wanted to obtain a confirmation of this result from his own instruments.

The introduction of the guard-ring surrounding the movable attracted disc got rid of a troublesome correction, which otherwise would have entered into the expression for the attraction between the discs. The reason for this is, of course, that the lines of electrostatic inductors are curved towards the edges of the discs thus :

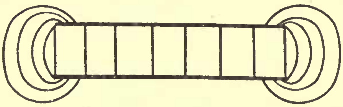

FIG. 3.—Curved lines of electrostatic induction.

Resistance.—Standards of resistance were always of importance since in cable construction high conductivity was of the greatest moment, so that it attracted attention at a very early date.

Ballistic Method.—If two coils having a mutual inductance and high resistance are connected as shown (Fig. 4), and a steady current passed through them, C_1 is then removed to reduce M to zero, and the inductive throw of the galvanometer noted, the battery still being in circuit. In this case it can be shown (*Abs. Meas.* p. 540, vol. ii.) that

$$R = M\frac{\pi \tan \theta_1}{T \sin \frac{1}{2}\theta_2},$$

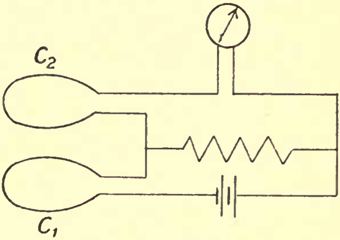

FIG. 4.—Absolute measurement of resistance.

where θ_1 is the steady deflection and θ_2 the induction kick in removing the coil.

Consequently the operation chiefly consists in determining M accurately. This method was due to Kirchhoff (*Poggen. Annalen*, t. 76, 1846).

The Earth Inductor Method.—Since it can be shown that a quantity of electricity passing through a ballistic galvanometer of the moving needle variety is given by

$$Q = \frac{H}{G} \cdot \frac{T}{\pi} \cdot \sin \tfrac{1}{2}\theta,$$

where H is earth's horizontal intensity,
T is periodic time,
G is galvanometer constant,
θ is angle of swing,

and since with an earth inductor rotated through 180° so as to cut the earth's lines of force due to this H, if the resistance of the inductor and galvanometer is R, then

$$\frac{2AH}{R} = \frac{H}{G} \cdot \frac{T}{\pi} \cdot \sin \tfrac{1}{2}\theta,$$

$$\therefore \quad R = \frac{2\pi GA}{T \sin \frac{1}{2}\theta}.$$

Other methods suggested included the damping method by Wiedemann who oscillated a magnet inside a coil with the circuit first open and then closed, and noted the damping in each case. This method is interesting, and will be dealt with below.

Revolving Coil.—The method suggested by Lord Kelvin of spinning a coil in the earth's magnetic field and observing the deflection, was used by the Committee of the British Association in their celebrated experiment of 1863. Afterwards the experiment was repeated by Lord Rayleigh and Professor Schuster (1881), and later by Lord Rayleigh with improved apparatus. This method, at first sight apparently simple, is in reality considerably complicated. Difficulties in carrying out the experiment arise from influence of air currents and measurement of speed and alterations of magnetisation of needle, besides eddy currents in the ring.

Revolving Disc Method of Lorentz.—This is probably the simplest method of measuring R in absolute units with great accuracy.

If a disc of metal be placed so that its axis is in the axis of a large solenoid in order that the field of force can be determined accurately, then if it be spun in this field it will act like Faraday's dynamo developing an E.M.F.

If, therefore, M is the number of lines cut by the disc when a unit current passes through the coil, then it cuts

$$n \,.\, \mathrm{M} \,.\, \mathrm{C}$$

when it revolves with n revolutions per second, and a current C passes through the solenoid.

If now the same current which energises the solenoid is allowed to flow through a resistance R, and further if the potential differences, viz. CR and nMC, are balanced potentiometrically, we have at once

$$\mathrm{R} = n\mathrm{M}.$$

Consequently, in this method an accurate determination of M, the mutual inductance between coil and disc, and an accurate determination of speed are required.

The connections used are shown in Fig. 5. The

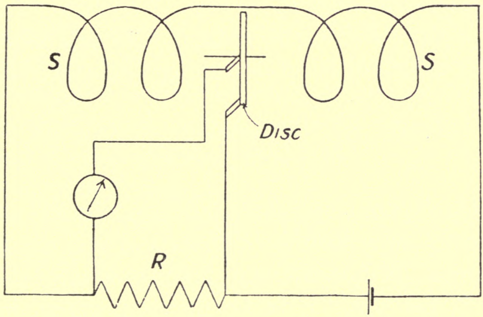

FIG. 5.—Method of Lorentz.

R, Resistance to be tested. SS, Solenoid.

diameter of the disc used was about 0·6 that of the coils, so that it moved in a constant field of force. Its edge was made cylindrical to take a brush which was amalga-

mated with mercury, together with the edge. Accuracy of balance was obtained when no deflection was obtained on reversing the battery current. A series of readings taken with a resistance R_1, a little less than R, and R_2 a little greater, gave galvanometer deflections on opposite sides of zero, and so R was determined by interpolation. Two series of results were obtained, one when the coils acting on the disc were close together, and one with them far apart. This was done because there was some distance where the rate of change of M with radius of coils was zero or the magnetic induction had a maximum value.

In calculating M again since the central brush touched a small area this had to be deducted. So far as M is concerned it is the mutual inductance coefficient of the helix and disc respectively, and represents the number of lines of induction passing through the disc when unit current passes through the helix.

For a helix and circle coaxial with one another we generally calculate M by beginning with the well-known integral

$$M = \iint \frac{\cos \epsilon}{r} ds_1 ds_2,$$

where ds_1, ds_2 are elementary portions of circle and helix respectively at a distance r apart and ϵ is the angle of inclination. If a_1 is radius of helix, a_2 that of the circle, θ_1 and θ_2 the angles between the radii drawn from the axis to the elements ds_1, ds_2 made with the plane through the axis and the radius of helix, then $\cos \epsilon = \cos (\theta_2 - \theta_1)$. If the pitch of the helix is p, then $p\theta_1$ is the distance between adjacent pitches parallel to the axis, so that

$$r = \sqrt{a_1^2 + a_2^2 - 2a_1a_2 \cos (\theta_1 - \theta_2) + p^2\theta^2},$$

the expression above for M transforms into

$$M=\int_0^{2\pi}\int_0^{\theta_0}\frac{a_1a_2\cos(\theta_1-\theta_2)d\theta_1d\theta_2}{\sqrt{a_1^2+a_2^2-2a_1a_2\cos(\theta_1-\theta_2)+p^2\theta_1^2}},$$

where θ_2 is the superior limit for θ.

This can be expressed in a series of definite integrals (see Gray, *Abs. Measurements*, p. 311, vol. ii.), and a series of computations leads to result that $M=k\,.\,n$ where k is a constant determined by calculation and by the use of tables.

For the small circle at the centre the field was assumed constant over it and M_0 was taken as

$$M_0=\frac{2\pi a_1^2}{(a_1^2+x^2)^{\frac{3}{2}}}\times\tfrac{1}{2}\pi\times(2{\cdot}096)^2,$$

a_1 being the radius of the solenoid, x is the distance between mean planes of coil and contact on disc – 2·096 radius of small area.

Joule's Method.—An interesting method was suggested by Joule and based on his result for the mechanical equivalent J for electric circuits.

Joule suggested that since

$$C^2Rt=JH,$$

$$\therefore\quad R=\frac{JH}{C^2t}.$$

This method does not, however, appear very acceptable, since it would require a very exact determination of "J" by calorimetric methods not susceptible of very great accuracy, and an error in measuring C would also be doubled in the denominator.

Results.—The result of all the preceding investigations referred to is that these quantities are defined as follows:—

An ampere is the unit of current and deposits 0·001118 grammes of silver per second from solution of silver nitrate. *An ohm* is the resistance of a column of mercury 1 millimetre in section and 106·3 cms. long at 0° C.

A volt is said to exist between the ends of a wire when the above current is passed through a wire of 1 ohm resistance.

Comparative Measurements

We now proceed to consider some of the methods adopted for measuring the different quantities occurring in the course of practical work.

Difficulties in comparative measurements arise from various causes and, generally speaking, methods suitable for measuring large quantities may not be equally suitable for measuring small ones. Another difficulty arises again if the current be an alternating one, or pulsating in character.

We shall consider now the comparative measurement of current.

Current.—If the current to be measured be a moderate one of a few amperes or some hundreds of amperes, any of the ammeters described later would suffice for its measurement. A fraction of the current passes through the working coils of the apparatus, and the scale is graduated to read the actual current passing on the circuit.

Large Currents.—If these are continuous and of the order of thousands of amperes, the current might be measured by making use of the volt drop in a low known resistance and observing the volt drop with a millvolt-

meter. Possibly the best way would be to balance the drop due to the current against that of a standard cell by the potentiometer, thereby finding the drop due to CR accurately and then equating

$$e = \mathrm{CR},$$

from which C can be found when R is known.

Very Small Currents.—These can be measured directly by means of a sensitive galvanometer easily to $1/100 \times 10^6$ ampere, provided the current is flowing steadily.

Alternating Currents. — For currents of ordinary magnitude there is no lack of measuring instruments.

These instruments without exception all measure

$$\mathrm{C}_m = \sqrt{\frac{1}{\mathrm{T}} \int_0^{\mathrm{T}} \mathrm{C}^2 dt},$$

the "square root of the mean square" of the periodic quantity.

If the current was a very large alternating one, it must be divided and measured by fractions.

If, again, it was very small, difficulties arise and some instrument such as a thermo-galvanometer may require to be used : Dr. Drysdale's alternating current potentiometer will be described in due course.

Instantaneous Currents.—Ordinary instruments give no indication of instantaneous currents. In order to detect them special instruments, such as oscillographs, or devices such as rotating contact maker of Joubert are used, when the current is periodic in character.

Volts.—Continuous current voltages are measured by many of the instruments as described later; also by

a potentiometer, the unknown voltage being balanced with that of a standard Clarke or Weston cell. This latter applies to very high and very low voltages.

Alternating Volts.—Instruments to read volts all measure the "root mean square" value.

Instantaneous Volts.—The same remarks apply here as to instantaneous currents, and, since $CR = e$, by passing the instantaneous current through a known resistance it may be measured.

Resistances.—Ordinary resistances can be measured by means of the Post Office Box. This covers a range usually from about $\frac{1}{100}$ ohm to several megohms.

When smaller than the lowest on Post Office Box, or small, say of the order $\frac{1}{5}$ to $\frac{1}{100}$ ohm, the potentiometer method is probably the most suitable.

When two similar coils of about a few ohms have to be compared, then Carey-Foster's method is usually adopted with a modified metre bridge.

For very low resistances again a "double" bridge of the Kelvin variety is required for great accuracy.

Alternating Resistance.—When an alternating current passes through a wire, it generally produces two effects: first, self-induction becomes active, and, secondly, there may be a "skin effect" if the wire is large in diameter or periodicity great. With current at 100 cycles per second a wire of 10 millimetres diar. of copper increases only about $\frac{1}{100}$ per cent in resistance, but if its diameter is increased to 22·4 millimetres the change in resistance is 17·4 per cent greater.

The self-induction of a straight wire is

$$L = \int\int \frac{ds \, . \, ds^1}{r} \cos\theta$$

(see A. Gray, *Absolute Measurements*, vol. ii. p. 293). This represents the coefficient for steady currents. For varying currents of sine form

$$l = \mathrm{L}\left\{\mathrm{A} + \mu\left\{\tfrac{1}{2} - \tfrac{1}{48}\frac{p^2 l^2 \mu^2}{\mathrm{R}^2} + \text{etc.}\right\}\right\},$$

where $p = \dfrac{2\pi}{n}$ (see Fleming, *Principles*, *etc.*, p. 131),

approximately $$l = \mathrm{LA} + \frac{\mathrm{R}^1}{p},$$

where R^1 is the resistance "ohmic" at high frequency.

For very high frequency, as used in wireless telegraphy, the resistance of the wire carrying the high frequency current may be found by comparing the rate of heat developed with that developed in a similar wire similarly situated carrying continuous current.

Capacity.—This for ordinary condensers may be measured by discharging it through a ballistic galvanometer. If the charging voltage is the same as that which gives a steady deflection through a certain resistance R, then

$$\mathrm{Q} = \mathrm{KV} = \frac{\mathrm{H}}{\mathrm{G}}\frac{\mathrm{T}}{\pi}\sin\tfrac{1}{2}\theta(1 + \tfrac{1}{2}\lambda),$$

$$\frac{\mathrm{V}}{\mathrm{R}} = \frac{\mathrm{H}}{\mathrm{G}}\tan a,$$

$$\therefore \quad \mathrm{K} = \frac{\mathrm{T} \,.\, \sin\tfrac{1}{2}\theta \,.\, (1 + \tfrac{1}{2}\lambda)}{\pi \mathrm{R} \tan a}.$$

Very Small Capacities.—As a rule some other method must be adopted if the capacity is not directly calculable, as in an air condenser. Advantage is taken of the fact

that if an intermittent current flows into the condenser, then

$$c = \mathrm{K}\frac{dv}{dt},$$

where $\frac{dv}{dt}$ is rate of change of voltage. By increasing $\frac{dv}{dt}$ a measurable current can be made to flow into the condenser.

Resonance methods also are used for this purpose, especially in wireless telegraphy work.

Capacity in Alternating Circuits.—When a main is on open circuit the capacity is sometimes measured by the capacity current, viz.

$$c = 2\pi n\mathrm{KV}$$

or

$$\mathrm{K} = \frac{c}{2\pi n\mathrm{V}};$$

as a rule n is about 50 cycles per second, and V may be some thousands of volts.

Self-Induction.—As a rule, in physical work the coefficient of self-induction is calculable from the shape of the circuit when this is of some simple form. In all other cases measurement is the only course, as the theory becomes extremely complex in most practical cases.

Self-Induction in Alternating Work.—In practice self-induction is associated with iron as a rule, and owing to the fact that the coefficient of permeability becomes meaningless to us, owing to the presence of hysteresis, L becomes quite incalculable in cases where iron is present. In practical engineering L is generally regarded as a constant quantity, and can be obtained in the case

of a choking coil by observing the current and voltage when the resistance is assumed from the formula

$$C = \frac{E}{\sqrt{r^2 + p^2L^2}}.$$

E and C are, of course, R.M.S. values.

Very Small Self-inductions.—These can be measured by making use of the resonance principle, and much the same remarks apply as to capacity.

Mutual Inductance.—Similar remarks apply in this case also. It will be seen later how by means of the vibration galvanometer these coefficients may be measured.

Magnetic Fields.—In the measurement again of very intense magnetic fields, such as in the air gap of a dynamo, various methods are adopted to ascertain its value.

Ballistic methods all depend on the fact that

$$\int \frac{dQ}{dt} dt = \frac{\int \frac{dN}{dt} dt}{R} = \frac{Nn_2}{R},$$

where n_2 are turns on search coil.

The Fluxmeter theory, which is independent of duration of discharge and resistance, will be discussed later.

The great advantage of ballistic testing consists in the fact that however weak the magnetisation may be, by increasing the turns on the exploring coil its effect can be magnified proportionately.

Other methods measure B by some effect, such as tractive effect, it produces.

And when obtaining very high inductions of the order

of 30,000-40,000 C.G.S. being per sq. cm., special methods of magnetisation, such as the Isthmus method of Ewing, have to be adopted, since it is almost physically impossible to magnetise iron by means of a solenoid alone up to this degree of saturation.

At the present time an instrument to measure magnetic qualities at very high inductions is required in practice, as magnetic properties of iron or iron alloys at moderate inductions appear to be no guide as to their properties at 30 to 40 thousand lines per sq. cm.

In view of its great simplicity, the method of testing by the magnetometer ought to be more used than at present.

Again, when extremely feeble magnetic forces or presence of iron in brass, etc., have to be compared, Curie's balance has to be used.

Electrolytic Resistance.—Ordinary methods of testing resistance generally break down in the presence of a back E.M.F.

The methods adopted are of two types only, viz. (1) Stroud and Henderson's method, (2) using an intermittent current and telephone or galvanometer.

In the former method two electrolytic cells are so arranged that their polarisation E.M.F.'s are opposed to one another, and as they are of different ohmic resistance, this difference of resistance is balanced by the arms of a bridge.

Kohlrausch's method, again, will be found minutely described in Whetham's *Theory of Solution.* It is pointed out by Lord Rayleigh in *Theory of Sound*, vol. i. p. 456, that in experiments involving the use of a telephone a frequency of 500-2000 is generally desirable for

sharpness of balancing. This agrees with the observations of the authors using the Kohlrausch method.

Scale Readings.—The above remarks are merely to draw the attention of the reader to the fact that in measuring any physical quantity, the method of doing it should be carefully considered.

A common application is finding the efficiency of a motor by measuring the input and brake horse power. Now it is quite clear that when the light load readings are being taken, a full load ammeter is not usually very suitable, as the scale deflection is small. This may lead to large errors in the lower readings.

Also wattmeters are frequently sold with a series resistance for the volt coil, the instrument being originally designed for a 100 volt circuit. Of course when the series resistance is used the deflection of the volt coil is reduced in proportion to the reduction of current. Consequently, if one is using a wattmeter to give a full scale deflection with say 10 kilowatts at 100 volts, it may not be much use for measuring fractions of kilowatts at 400 or 500 volts.

It seems so obvious an error that it is almost absurd to allude to it here, and yet it occurs so frequently in practice that we offer no excuse for referring to it as an example of the simple way in which errors may easily arise.

Power.—Again, in measuring, say the brake horse power of a motor, the accuracy of the brake readings should be taken into consideration. There is in no case any necessity for making elaborately exact determinations of the input, say, of a motor when there is a 5 per cent error in the brake measurements. Yet one frequently sees reference to "92·5 per cent efficiency."

This would mean, of course, an accuracy of measurement to about 1 in 1800.

Alternating Power.—Many of the methods of measuring alternating power may be unsuitable under the conditions in practice. For instance, the 3 ammeter or 3 voltmeter methods are only rarely suitable. Again, with high electromotive forces and low power factors, great care has to be exercised to avoid error in making efficiency determinations.

These remarks apply with still greater force to determinations of "over all" efficiency of engines and dynamos. Attention should be directed to the accuracy of the I.H.P. indications as well as to the electrical measurements.

Particular Types of Instruments.—Certain classes of instruments again, unless they are working at the top of their range of scale readings, are not suitable for measuring small quantities.

Those involving a "square law" suffer from this defect. If R is the reading, then we have

$$\mathrm{R} \propto \mathrm{C}^2 \quad \text{say,}$$

$$d\mathrm{R} \propto 2\mathrm{C}d\mathrm{C},$$

so that

$$\frac{d\mathrm{R}}{\mathrm{R}} \propto \frac{2d\mathrm{C}}{\mathrm{C}},$$

so that, besides the accuracy diminishing with the variable C, the error is doubled.

As a rule, in such instruments the scale divisions towards the zero end are cramped, increasing the error dC still further for the lower readings.

Other methods of measuring power may be dependent

on the assumption of, say, a sine wave having been made in proving the original formulae which is used.

Again, frequency may cause errors in alternating work and necessitate correcting factors being used with certain instruments.

The matters shall be considered in detail in later chapters, and examples of some of the difficulties met with illustrated.

Absolute Measurements

Dimensions.—All the units with which we have to deal are derived from the so-called fundamental units of Length, Mass, and Time.

Of these three quantities, it may be observed, we know nothing *absolutely*, but their existence is generally assumed. All our ideas of these quantities are relative, yet on them the whole of dynamics is founded.

From these we find that

$$\text{Velocity} = \frac{\text{L}}{\text{T}}, \quad \text{Acceleration} = \frac{\text{L}}{\text{T}^2},$$

since $a = \frac{d\text{V}}{dt}$, and force being defined by mass × acceleration, we have

$$\text{F} = \text{M}\frac{d\text{V}}{dt} = \text{MLT}^{-2}.$$

These are generally known as the derived mechanical units, and we could, in the same way, obtain

$$\text{Work} = \text{F} \times \text{L} = \text{ML}^2\text{T}^{-2},$$

and since Power or Activity $=\frac{dW}{dt}$,

we have

$$\text{Power}=ML^2T^{-3}.$$

Now these expressions give what is known as the "dimensions" of the quantities, and if any formula for a force has dimensions other than those above, it must be wrong; apart, therefore, from the necessity for having absolute units as a starting-point, knowing their dimensions is valuable in practice for testing formulae by inspection.

Electrical work is further complicated by the fact that we have two systems of units, and since the action of instruments is dependent on both electrostatic and electromagnetic forces, a brief explanation is necessary.

As regards the fundamental starting-point, the units of length, mass, and time were agreed upon as being the centimetre, gramme, and second.

If we wish to measure a current absolutely, we may think of it either as producing a given magnetic force at, say, the centre of a turn of wire of some unit value, or we may think of it as $\frac{dQ}{dt}$, the passage of a given quantity of electricity per second through the wire.

Assuming the Inverse square law we have

$$\frac{mm}{d^2}=F$$

as an electromagnetic starting-off point, where mm are the quantities of magnetism, d their distance apart, F the force of attraction or repulsion;

consequently $\qquad m=d\sqrt{F}.$

Now the dimensions of F given above are MLT^{-2}; therefore the dimensions of m are

$$L\sqrt{MLT^{-2}}$$

or
$$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}.$$

Now the force in electrostatic measure depends on

$$\frac{qq}{d^2}=F,$$

so that the dimensions of q are the same as those for a quantity of magnetism, viz.

$$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}.$$

If, therefore, on the electrostatic system, we define a current as Quantity/Time, we obtain for its dimensions

$$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}.$$

If we wish to define the current in electromagnetic measure we say that a "unit" current is one which produces unit magnetic force at unit distance. Unit magnetic force is that force which, acting on a gramme of matter, produces unit acceleration in unit time.

Ampère has shown that the magnetic effect of an element of current on a magnet pole of strength m when the angle is 90° is given by

$$dF=\frac{C\,ds\,m}{d^2},$$

where d is distance.

Consequently we have for current

$$C=\frac{dF\,.\,d^2}{m\,.\,ds}.$$

The dimensions of dF are MLT^{-2} and d^2 is of course L^2, m is $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$, and the small elementary length is L. Hence the dimensions of C in electromagnetic units are

$$C = \frac{MLT^{-2}L^2}{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}L} \text{ or } M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}.$$

Consequently we see that the ratio current in electrostatic units to current in electromagnetic units is

$$\frac{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}}{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}},$$

or

$$\frac{L}{T}, \text{ a velocity.}$$

This velocity is generally known as "v," and it has been proved by Maxwell that this quantity equals the velocity of light, or 3×10^{10} cms. per second (see p. 44).

We have referred to this question of current at some length because forces due to currents are important instrumentally, and one of the earliest methods of finding "v" used by Clerk Maxwell was to balance a force due to current by an equal electrostatic force.

If two currents C_1C_2 be allowed to charge two spheres at a distance r apart, and K is the specific inductive capacity, t the time of charge, we have for the electrostatic force

$$\frac{C_1C_2t^2}{Kr^2}.$$

Now the force due to an infinitely long current C_1 on an element of current C_2 in another conductor is

$$\frac{2\mu C_1C_2ds}{r},$$

so that the force on a length $\frac{r}{2}$ is

$$\frac{2\mu C_1 C_2}{r}\int_0^{\frac{r}{2}} ds$$

or $$\mu C_1 C_2,$$

where μ is the permeability of the medium. Hence again, we have, if these quantities are equal,

$$\frac{1}{\sqrt{\mu K}} = \frac{r}{t}, \text{ a velocity.}$$

We have already found the dimensions of current. Consider next electromotive force. In this case, if we have a unit charge of electricity forced from an infinite distance against the repulsion of a quantity of electricity q on a sphere, the work done will be

$$\int_r^{\infty} \frac{q}{r^2} = \frac{q}{r},$$

where r is the radius of the charged sphere. The dimensions then of Potential are $\frac{Q}{L}$ in electrostatic measure,

or $$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}.$$

Now in electromagnetic measure potential is defined as work done in moving a unit of electricity against a difference of electric potential.

Consequently if V denotes this difference of potential,

$$VQ = \text{work},$$

$$V = \frac{\text{work}}{Q} = \frac{ML^2T^{-2}}{M^{\frac{1}{2}}L^{\frac{1}{2}}},$$

$$V = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2},$$

on the electromagnetic system.

Quantity on the electrostatic system has already been obtained on the electromagnetic system $Q = Ct$, and since current has dimensions $L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$,

$$Q = M^{\frac{1}{2}}L^{\frac{1}{2}}.$$

Resistance is defined by the ratio of electromotive force to current, so on the electromagnetic system its dimensions are

$$\frac{L^{\frac{3}{2}}T^{-2}M^{\frac{1}{2}}}{L^{\frac{1}{2}}T^{-1}M^{\frac{1}{2}}} \text{ or } \frac{L}{T},$$

and on the electrostatic system

$$\frac{L^{\frac{1}{2}}T^{-1}M^{\frac{1}{2}}}{L^{\frac{3}{2}}T^{-2}M^{\frac{1}{2}}} \text{ or } \frac{T}{L}.$$

Hence R electromagnetic $= v^2 \times r$ electrostatic.

Self and Mutual Inductance.—Since E.M.F. is generated when lines of force are cut, we have

$$e = L_0\frac{dC}{dt} \text{ or } M\frac{dC}{dt}.$$

Consequently in electromagnetic units

$$L_0 = \frac{e}{\frac{dC}{dt}} \text{ and } M = \frac{e}{\frac{dC}{dt}}$$

dimensionally. That is

$$\frac{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}}{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}}=L,$$

so that self and mutual inductance are both lengths on the electromagnetic system.

The other important dimension is *Intensity* of magnetic fields. This is given by $\frac{m}{\text{area}}$, and since the dimensions of $m = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$, dividing by L^2 gives $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$ as dimensions of I.

The other quantity is B, and since

$$B = 4\pi I + H,$$

the dimensions of H are $\frac{\text{Current}}{\text{Length}}$ or $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$, and we see that this is of the same dimensions as I, consequently B and I are of the same dimensions.

It is sometimes useful to note that since $I = \frac{m}{a}$,

$$B = 4\pi\frac{m}{a} + H,$$

which allows one to change from charges on poles to fluxes and *vice versa*. The various dimensions are given in any text-book of Physics.

We now proceed to consider the methods adopted to actually measure a few of these quantities in absolute units, and in the space at our disposal we only refer to one or two methods.

We have selected those methods which appeal most to us from the instrument-maker's point of view.

The Quantity "v"

It having been shown, as already mentioned, that the change ratio between the electrostatic and electromagnetic system of units depended upon "v," many attempts have been made to measure it accurately, and compare the result obtained with the measured velocity of light by more direct methods.

The student is, of course, aware that the actual velocity of light was determined by the Danish philosopher Roëmer, from eclipses of Jupiter's satellites in opposition and conjunction, as early as 1675, and has since been measured by various other methods. The chief methods are those of Astronomical Aberration, and the toothed wheel eclipse method due to Fizeau and Foucault.

Since Maxwell has shown theoretically that the velocity of an electric impulse through space was given by the ratio "v" occurring in ratio of Electrostatic and Electromagnetic units, it became of importance to measure it accurately. We shall, therefore, refer to a few methods very briefly.

The first attempt was by Weber and Kohlrausch in 1856, and consisted in discharging a quantity of electricity from a Leyden jar through a ballistic galvanometer. The voltage of the jar was known in electrostatic units and its capacity in electrostatic units from measurement.

The same method on a more refined scale was utilised by Rowland, using an instrument of the Kelvin Absolute Electrometer type to measure the potential.

Kelvin compared the potential drop in a wire of

resistance absolute R, by measuring C absolutely and by measuring V electrostatically with an absolute electrometer, then

$$\text{“}v\text{”} = \frac{\mathrm{CR}}{\mathrm{V}}.$$

Maxwell's method already referred to is by balancing the electromagnetic repulsion of two coils by electrostatic forces.

Again, if an air condenser is made, its electrostatic capacity can be calculated exactly by the formula

$$\mathrm{K} = \frac{\mathrm{A}}{4\pi t}$$

in electrostatic units, where A is the area of plate, t their distance apart. The condenser must have a guard-ring to render the above formula exact. This capacity is then measured electromagnetically by a ballistic galvanometer carefully corrected for damping, then it can be shown

$$v = \sqrt{\frac{\mathrm{K}_1}{\mathrm{K}_2}},$$

K_1, K_2, electrostatic and electromagnetic capacity respectively.

Loss of Charge Method.—If a condenser is allowed to discharge through a high resistance R we have

$$\mathrm{K}\frac{d\mathrm{V}}{dt} + \frac{\mathrm{V}}{\mathrm{R}} = 0$$

for equation at any time t, KV being in electrostatic

units. Suppose original V is halved in time t, then it follows that

$$R = \frac{t}{K \log_{\epsilon} 2}.$$

If now K, R are the electromagnetic values of capacity and resistance, we have

$$v = \sqrt{\frac{RK \log_{\epsilon} 2}{t}}.$$

The determination of velocity of light by Newcomb using Foucault's method in 1888 makes it $2{\cdot}998 \times 10^{10}$ and J. J. Thomson and Searle's determination of "v" in 1890 is $2{\cdot}995 \times 10^{10}$.

Relation between Electrostatic and Electro-Magnetic Units

Electrostatic Unit of	Electromagnetic Unit of
$v \times$ E.M.F.	E.M.F.
current	$v \times$ current
$v^2 \times$ resistance	resistance
capacity	$v^2 \times$ capacity
quantity	$v \times$ quantity

CHAPTER II

DAMPING

Electromagnetic Damping.—This form of damping being of the greatest importance, we shall consider it in the first place with reference to a moving coil galvanometer.

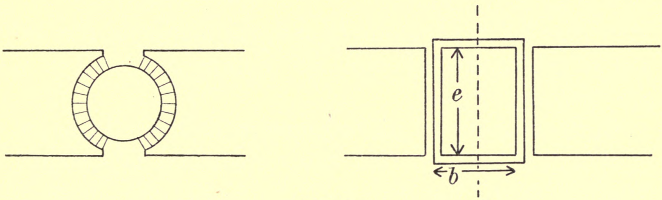

FIG. 6.

Let the galvanometer be of the general type shown in Fig. 6, in which a coil is moving in a field of radial and uniform intensity H in C.G.S. units and having n turns.

Force on coil in dynes is $\mathrm{H}lc2n$ maximum, and the torque is $\mathrm{H}lnbc$, where c is the current passing. Put $\mathrm{G}=\mathrm{H}lnb$, then torque is $\mathrm{G}c$.

If the angular velocity of the coil is ω at any moment, the electromotive force developed is

$$\frac{2ln\mathrm{H}b\omega}{2},$$

or

$$\mathrm{G}\omega.$$

If this E.M.F. is allowed to send a current through a resistance R in the circuit (including the resistance of the coil), then

$$c = \frac{e}{\mathrm{R}} \text{ at any instant,}$$

or
$$c = \frac{\mathrm{G}\omega}{\mathrm{R}}.$$

But it has been shown above that the torque due to any current is Gc; therefore

$$\text{Torque} = \frac{\mathrm{G}^2\omega}{\mathrm{R}}.$$

The other torques tending to stop the motion of the coil will be mechanical friction, air friction, and torsion of the suspension.

In the majority of galvanometers mechanical friction is absent altogether, and the air friction is assumed proportional to the velocity. Physically the effect of air friction is to stop the motion by creating eddies in the air. The problem of a sphere oscillating in air has been worked out, and when it oscillates in air the corrections are in two terms, one, an increase of inertia due to air moved with the sphere greater than a half of the fluid displaced by the sphere, and another term a frictional force varying as the velocity (see *Hydrodynamics*, Lamb, p. 584). The first term may be omitted in this case since the mass of the air displaced by the moving coil is exceedingly small. The term due to torsion is, of course, proportional to θ with a monofilar suspension.

Hence we have by the well-known equation :

Moment of Acceleration = Moment of Forces,

or $$-\mathrm{I}\frac{d^2\theta}{dt^2} = \sum \text{(retarding torques)}.$$

The retarding torques are as follows :—

(1) Due to damping current $= \dfrac{\mathrm{G}^2\omega}{\mathrm{R}}$.

(2) Air friction $= k_2\omega$.

(3) Torsion of suspension $= k_1\theta$.

Putting $\dfrac{d\theta}{dt} = \omega$, we have for the equation of motion

$$\frac{d^2\theta}{dt^2} + \left(\frac{\mathrm{G}^2}{\mathrm{RI}} + \frac{k_2}{\mathrm{I}}\right)\frac{d\theta}{dt} + \frac{k_1}{\mathrm{I}}\theta = 0.$$

Putting $$\frac{\mathrm{G}^2}{\mathrm{RI}} + \frac{k_2}{\mathrm{I}} = a, \quad \frac{k_1}{\mathrm{I}} = b,$$

then $$\frac{d^2\theta}{dt^2} + a\frac{d\theta}{dt} + b\theta = 0.$$

A. *Ballistic Galvanometer.*—If the damping is small, a in above equation $= 0$, so that our equation becomes

$$\frac{d^2\theta}{dt^2} = -b\theta.$$

Hence $$\theta = \mathrm{A}\sin\sqrt{b}t + \mathrm{K}.$$

Now when $$\theta = 0,\ t = 0, \text{ and } \mathrm{K} = 0,$$

when $$t = \frac{\mathrm{T}}{4},\ \sqrt{b}t = \frac{\pi}{2}, \text{ and } \sin\sqrt{b}.t = 1,$$

or suppose now at any instant c is the current passing, then if this is a transient current, we have

$$G\int_0^T c\,dt = I\frac{d\theta}{dt},$$

or $I\omega_0$, where ω_0 is the initial velocity.

Now $$\frac{d\theta}{dt} = \frac{d}{dt}\left(\theta_1 \sin\sqrt{\frac{k_1}{I}}t\right),$$

when $$\frac{d\theta}{dt} = \theta_1\sqrt{\frac{k_1}{I}}\cos\sqrt{\frac{k_1}{I}}t,$$

$$t=0,\ \cos\sqrt{\frac{k_1}{I}}t = 1,$$

$$\omega_0 = \theta_1\sqrt{\frac{k_1}{I}},$$

$$\therefore \quad GQ = I\omega_0 = \frac{\theta_1 Tk}{2\pi},$$

or $$Q = \frac{k}{G}\cdot\frac{T}{2\pi}\cdot\theta_1,$$

where θ_1 is the "throw."

Example.—If a steady current was passed through the ballistic coil the deflection would be

$$c = \frac{k}{G}\theta_2,$$

and if we compare Q with c where c is the steady current we eliminate both H and G, and obtain

$$\frac{Q}{c} = \frac{T\theta_1}{2\pi\theta_2}.$$

A condenser $k=10^{-6}$ farads was discharged through the coil after being charged with 2·0 volts, $\theta_1=61$ divisions, $T=4{\cdot}48$ seconds. The same E.M.F. applied through $9{\cdot}22\times10^5$ ohms gave 47 divisions.

Now $$\frac{T\theta_1}{2\pi\theta_2}=0{\cdot}927,$$

and $$\frac{Q}{c}=\frac{E10^{-6}}{\dfrac{E}{9{\cdot}22\times10^{-5}}}=0{\cdot}922.$$

We see, therefore, that for an undamped galvanometer the agreement was very close.

B. *Grassot Fluxmeter* (*vide* p. 65).—Suppose next that there is no torsional control or that in one equation $b=0$.

Then $$\frac{d^2\theta}{dt^2}=-a\frac{d\theta}{dt},$$

$$\therefore \quad \frac{d\theta}{dt}=A\epsilon^{-at}.$$

Let $$\omega_0=\frac{d\theta}{dt} \text{ when } t=0,$$

then $$\frac{d\theta}{dt}=\omega_0\epsilon^{-at}.$$

Hence $$\theta=\frac{1}{a}\omega_0\epsilon^{-at}+K,$$

if when $$t=0,\ \theta=0,\ K=\frac{1}{a}\omega_0,$$

$$\therefore \quad \theta=\frac{\omega_0}{a}(1-\epsilon^{-at}).$$

For maximum deflection $t = \infty$, then

$$\theta_{\max} = \frac{\omega_0}{a} = \frac{\omega_0}{\frac{k_2}{\mathrm{I}} + \frac{\mathrm{G}^2}{\mathrm{RI}}}.$$

Let $$\frac{k_2}{\mathrm{I}} = 0, \text{ then } \theta_{\max} = \frac{\omega_0 \mathrm{RI}}{\mathrm{G}^2}.$$

Suppose the "throw" to be obtained from a search coil,

$$\mathrm{Q} = \frac{\text{Line turns}}{\mathrm{R}}.$$

But $$\mathrm{GQ} = \mathrm{I}\omega_0,$$

$$\therefore \quad \theta_{\max} = \frac{\mathrm{I}\omega_0 \mathrm{R}}{\mathrm{G}^2},$$

and substituting for ω_0

$$\theta_{\max} = \frac{\mathrm{GQR}}{\mathrm{G}^2}.$$

But $$\mathrm{Q} = \frac{\text{Line turns}}{\mathrm{R}},$$

$$\therefore \quad \theta_{\max} = \frac{\text{Line turns}}{\mathrm{G}}.$$

It will be noticed this result is altogether independent of R. Also it follows that the larger the value of G the smaller will be the maximum deflection, *i.e.* for small values of line turns the movement must be insensitive.

The advantage of a large value of G is that effect of friction of air, mechanical friction and torsion are entirely eliminated.

Test of Fluxmeter.—The search coil was placed in the air gap of an electromagnet, and a constant exciting current which passed through the magnet coils was made and broken, the value of the current being 0·3 amperes. The resistance in series with the fluxmeter coil and movement was varied as follows :

Resistance of coil, 8·5 ohms. Resistance of movement, 2·2 ohms.

Added Resistance.	Deflection.	Remarks.
0	125	Mean values only.
10	125	
20	124	
50	121	
100	115	
200	104	

This table enables one to judge within what limits the resistance in series with the coil may be varied without introducing errors.

The instrument was also tested with continuous current and

$$C = 6 \times 10^{-7} \text{ in amperes per division.}$$

The time of a complete oscillation was about 15 seconds.

The total angle of scale $= 2 \sin^{-1}\left(\frac{7}{11}\right)$, or $2 \times 39^\circ : 4'$.

Each division is therefore about $0{\cdot}4^\circ$.

General Case of Damping.—

$$\frac{d^2\theta}{dt^2} + a\frac{d\theta}{dt} + b\theta = 0.$$

If $$\frac{a^2}{4} - b \text{ is } +\text{ve},$$

then $$\theta = \mathrm{A}\epsilon^{m_1 t} + \mathrm{B}\epsilon^{m_2 t},$$

$$m_1 = -\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}, \qquad m_2 = -\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}.$$

In Grassot's fluxmeter when $t=0$, $\theta=0$,

$$\therefore \quad \mathrm{A} + \mathrm{B} = 0.$$

$$\frac{d\theta}{dt} = \mathrm{A}m_1\epsilon^{m_1 t} + \mathrm{B}m_2\epsilon^{m_2 t}, \text{ and when } t=0, \ \frac{d\theta}{dt} = \omega,$$

$$\therefore \quad \omega_0 = m_1\mathrm{A} + m_2\mathrm{B}.$$

$$\mathrm{A} = -\mathrm{B} = \frac{\omega_0}{m_1 - m_2},$$

$$\theta_{\max} = \frac{\omega_0}{m_1 - m_2}\epsilon^{m_1\mathrm{T}}\left(\frac{m_2 - m_1}{m_2}\right),$$

$$= \frac{\omega_0}{-m_2}\epsilon^{m_1\mathrm{T}},$$

and $$\mathrm{T} = \frac{1}{2\beta}\log_\epsilon \frac{m_2}{m_1}.$$

N.B.—At end of swing

$$\theta = \theta_{\max}, \ \omega = 0, \text{ and } t = \mathrm{T},$$

$$\therefore \quad 0 = m_1\epsilon^{m_1\mathrm{T}} - m_2\epsilon^{m_2\mathrm{T}},$$

$$= \epsilon^{a\mathrm{T}/2}(m_1\epsilon^{\beta\mathrm{T}} - m_2\epsilon^{-\beta\mathrm{T}}),$$

$$\therefore \quad \epsilon^{m_2\mathrm{T}} = \frac{m_1}{m_2}\epsilon^{m_1\mathrm{T}}.$$

Putting

$$\beta=\sqrt{\frac{a^2}{4}-b}, \text{ we have } \epsilon^{2\beta \mathrm{T}}=\frac{m_2}{m_1}, \text{ and } \mathrm{T}=\frac{1}{2\beta}\log_\epsilon \frac{m_1}{m_2}.$$

Galvanometer Oscillations.—

$$\frac{a^2}{4}-b \text{ is } -\text{ve},$$

then $$\theta=(\mathrm{C}\sin\beta t+\mathrm{D}\cos\beta t)\epsilon^{-at/2},$$

$$\beta=\sqrt{b-\frac{a^2}{4}}.$$

If when

$$t=0, \quad \theta=\theta_{\text{max}},$$

$$\mathrm{D}=\theta_{\text{max}}.$$

Again, when $$t=\frac{\mathrm{T}}{4}, \quad \beta t=\frac{\pi}{2}, \theta=0,$$

then $$\mathrm{C}=0,$$

so that $$\theta=\theta_{\text{max}}\epsilon^{-at/2}\cos\beta t.$$

Since $$\beta\frac{\mathrm{T}}{4}=\frac{\pi}{2}, \text{ hence } \mathrm{T}=\frac{2\pi}{\beta}=\frac{2\pi}{\sqrt{b-\frac{a^2}{4}}},$$

or damping *increases* periodic time.

Tests on Galvanometer.—A galvanometer was employed having two coils, one the ordinary working coil (1), the other (2) arranged for damping. A current was passed through it and a steady deflection was obtained. The circuit key was then opened and the oscillations observed.

Terminals on Open Circuit.—Resistance of coil No. 2, 5·5 ohms. Current to produce 1 m/mm. deflection at 190 cms., 10^{-7} amperes. Deflection, 43 m/mms. $E = 2·15$ volts, $R = \frac{1}{2}$ megohm. Successive throws were :

No. 1.

Left.	Calculated.	Right.	Calculated.	Remarks.
		90	...	Time for throw to become 9 m/mms., 86 seconds—17 complete oscillations. Periodic time $= \frac{86}{17} = 5·06$ seconds.
84	83·8			
		78	78	
72	72·6			
		67·5	67·6	
62	62·9			
		58	58·6	
54·5	54·5			
		51	50·8	
48	47·3			
		44	...	
41	41			
		38	38·1	
36	35·5			
		33	33	
31	30·8			
		28	28·6	
26	26·7			
		24·5	24·9	
23	23·1			
		21·5	21·5	
20	20			
		19	18·6	
18	17·4			
		17	16·2	

The same coil was then short circuited, and the following was the result :

No. 2.

Left.	Right.	Remarks.
	90	Time to come to rest, 7 seconds.
13·5		
	1·5	Periodic time $\frac{7}{1 \cdot 25}$
0		$= 5 \cdot 6$ seconds.

No. 3. With 20 ohms in series $5\frac{1}{4}$ periods were observed in 27·4 seconds, therefore the periodic time was 5·22 seconds.

No. 4. With 200 ohms it was 5·07 seconds.

The other coil, No. 1, was then tested in the same way. Its resistance was 35·4 ohms. Current per m/mm., $1 \cdot 12 \times 10^{-8}$ ampere.

No. 1. Coil short circuited :

Reading.	Time.	Remarks.
	Seconds.	
90	0	No oscillations.
60	3·2	
40	7·6	
30	10	
20	14	
10	20	
0	40·6	

No. 2. With 2000 ohms in series the readings were as follows :

Left.	Right.	Remarks.
	90	Time of 5·75
60		periods, 30 secs.
	40	Periodic time =
27		5·22 seconds.
	18	
11		
	8	
6		
	4	
3		
	2·5	
1·5		
	0	

This agrees very closely with the test for the other coil, viz. No. 3, and any difference which arises is probably due to the windings on the coil, the dimensions and air gap being the same for both coils.

Comparison of $\frac{G^2}{R}$ *for the Coils.*—Since for coil (1) $G_1 \times 1{\cdot}12 \times 10^{-8} = 1$ m/mm. deflection,

$$G_1 = \frac{k}{1{\cdot}12} \text{ and } R_1 = 2035{\cdot}4\,;$$

coil (2) $$G_2 = \frac{k}{10} \quad R_2 = 25{\cdot}5,$$

we have $$\frac{G_1^2}{R_1} = \frac{k^2}{1{\cdot}12^2 \times 2035{\cdot}4},$$

$$\frac{G_2^2}{R_2} = \frac{k^2}{100 \times 25{\cdot}5}.$$

The ratio is then

$$\frac{2550}{2543} \text{ approximately,}$$

so that apparently

$$\frac{G_1^2}{R_1} \doteqdot \frac{G_2^2}{R_2}.$$

Critical Resistance.—Referring again to the general equation

$$\frac{d^2\theta}{dt^2} + a\frac{d\theta}{dt} + b\theta = 0,$$

we see that the condition that the motion will be oscillatory, or non-oscillatory, depends on whether the roots of the auxiliary quadratic equation,

$$m^2 + am + b = 0,$$

are imaginary or real. Therefore for non-oscillatory motion $\frac{a^2}{4} - b$ is positive, and for oscillatory motion $\frac{a^2}{4} - b$ is negative. Hence there is a value of R which makes the motion just cease to be oscillatory. This is easily seen to be given by

$$b = \frac{a^2}{4} \text{ or } \frac{k_1}{I} = \frac{\left\{\frac{G^2}{RI} + \frac{k_2}{I}\right\}^2}{4}$$

Neglecting the air damping term this reduces to

$$\frac{4\pi}{T} = \frac{b^2H^2(2bln)^2}{IR},$$

$$\therefore \quad R = \frac{4b^4l^2n^2H^2T}{4\pi I},$$

or
$$R = \frac{b^4 l^2 n^2 H^2 T}{\pi I}.$$

Remark on General Equation.—In deducing the general equation the torque due to the induced current in the coil was taken as

$$Hlnbc;$$

this, of course, neglects the diminution due to any self-induction. If the self-induction of the coil was L, then the torque would be

$$Hlnbc - cL\frac{dC}{dt}.$$

Generally speaking, the latter term is negligible, and so it has not been taken into account in our equation.

Air Damping.—Galvanometers such as Lord Kelvin's mirror type were damped solely by air, and when electromagnetic damping was used the coil of the galvanometer was short circuited by means of a key.

Omitting then the electromagnetic term in the equation we obtain for a small magnet in the earth's field moving with air damping only, the expression

$$\frac{d^2\theta}{dt^2} + \frac{k_2}{I}\frac{d\theta}{dt} + \left(\frac{k_1}{I} + \frac{MH}{I}\right)\theta = 0.$$

The torque due to the earth's field acting in the little needle is

$$2\lambda m H \sin\theta$$
$$= MH \sin\theta,$$

but since θ is generally small, we may replace $\sin\theta$ by θ. This is of the same form as previously, viz.:

$$\frac{d^2\theta}{dt^2} + a\frac{d\theta}{dt} + b\theta = 0,$$

where $$a=\frac{k_2}{I}, \text{ and } b=\left(\frac{k_1}{I}+\frac{MH}{I}\right).$$

As in the former case we solve the auxiliary equation for oscillations and find

$$T=\frac{2\pi}{\sqrt{\frac{MH}{I}+\frac{k_1}{I}-\frac{k_2^2}{4I^2}}}.$$

We see therefore that

$$T=2\pi\sqrt{\frac{I}{MH+k_1-\frac{k_2^2}{4I}}},$$

which shows that damping increases the periodic time, and that torsion of the fibre diminishes it, also that a large amount of inertia eliminates the effect of air friction. Without damping or torsion, which previously was the rule with these galvanometers used ballistically, we have

$$T=2\pi\sqrt{\frac{I}{MH}}.$$

It will be noticed that this is the ordinary form for periodic time of an oscillating magnet used as in "H" determinations. In this case, as is usual, torsion and air friction are entirely eliminated.

In the older galvanometers of the moving needle variety the torsional control was generally very small. Joule's improved galvanometer was wound with 2798 turns of No. 40 silk covered copper wire on a wooden reel 4 inches in diameter, the suspension was $1\frac{1}{2}$ inches

long of unspun silk fibre, and the needle was only $\frac{1}{4}$ inch long. The torsional control gave only 1° of torsion in 360° of twist, and the air damping stopped the motion of the needle in 12 seconds (*vide* Joule's *Scientific Papers*, vol. i. p. 405).

Again, in the galvanometer which Joule used in his determination of the "Dynamical Equivalent of Heat from the Thermal Effects of Electric Currents" (*B.A. Report*, 1867), 360° of twist gave only 3·5 minutes of torsion. This galvanometer was used in series with a "current weigher."

Determination of R *absolutely by Damping.*—Weber oscillated a magnet inside a coil and noted the logarithmic decrement on open and closed circuit respectively. In the former case the equation was the simple one just referred to for air damping, in the latter it included the electromagnetic damping term and was further corrected for electromotive force generated on the coil, and the self-inductive action of the coil. By this means the value of R is obtained in terms of measurable constants (*vide Absolute Measurements*, p. 557, vol. ii.).

Comparison of Resistances by Damping.—Let λ_1 be the logarithmic decrement when the galvanometer is on open circuit,

λ_2 the same when short circuited,
λ_3 the same when R_1 is on circuit,
λ_4 the same when R_2 is on circuit.

Then it can be shown that

$$\frac{R_1}{R_2} = \frac{\lambda_2 - \lambda_3}{\lambda_2 - \lambda_4} \cdot \frac{\lambda_4 - \lambda_1}{\lambda_3 - \lambda_1}.$$

Hence if R_2 is known R_1 can be found. In this case the galvanometer should be of low resistance, and only slightly damped, and, in order to obtain readable differences in R_1, R_2, must not be too great, or the method breaks down.

Periodic Time.—Writing our equation for damping in the form

$$\frac{d^2\theta}{dt^2}+a\frac{d\theta}{dt}+b\theta=0,$$

the imaginary roots of the auxiliary equation are

$$-\frac{a}{2}\pm\sqrt{\frac{a^2}{4}-b}.$$

Let these equal $\quad -\alpha\pm\beta i,$

then $\quad \alpha^2+\beta^2=b.$

But the period of oscillation is $T=\dfrac{2\pi}{b}$, and of a damped oscillation, $T=\dfrac{2\pi}{\beta}$.

$$\frac{T^2}{T_0^{\,2}}=\frac{b^2}{\beta^2}=\frac{\alpha^2+\beta^2}{\beta^2},$$

$T>T_0$, the period for a damped vibration is increased.

Effect of Small Amount of Damping.—It is easily shown that if the damping is small it merely increases the periodic time by a second order quantity.

If T is the damped period, T_0 the undamped period, and λ the logarithmic decrement, then we have

$$T^2=T_0^{\,2}\left(1+\frac{\lambda^2}{\pi^2}\right),$$

and expanding by the Binomial Theorem, we obtain

$$T = T_0\left(1 + \tfrac{1}{2}\frac{\lambda^2}{\pi^2}\right) \quad \text{approximately.}$$

λ is generally small, so that $\dfrac{\lambda^2}{\pi^2}$ is a small quantity.

This will be further considered in the section on Logarithmic Decrement.

Calculation of Maximum Throw with a Given Total Resistance.—With the same notation as above let

$$\frac{a}{2} = \alpha, \quad \sqrt{\frac{a^2}{2} - b} = \beta,$$

then $$\theta_{\max} = -\frac{\omega_0}{m_2}\epsilon^{m_1 T}.$$

But $$GQ = I\omega_0,$$

$$\therefore \quad \frac{GM}{R} = I\omega_0.$$

M represents "line turns."

Hence $$\omega_0 = \frac{GM}{RI} = \frac{G^2 M}{RIG},$$

or $$\theta_{\max} = \frac{-\dfrac{G^2}{RI}}{m_2}\frac{M}{G}\epsilon^{m_1 T}.$$

Putting $$x = \frac{-G^2/RI}{m_2}\epsilon^{m_1 T},$$

$$\theta_{\max} = \frac{M}{G}x,$$

or
$$\theta_{\max} = \frac{-2a}{m_2}\epsilon^{m_1 \mathrm{T}} = \frac{-2a}{m_2}\left(\frac{m_2}{m_1}\right)^{\frac{m_1}{2\beta}}.$$

Now let
$$\epsilon^{m_1 \mathrm{T}} = y, \quad \log_\epsilon y = m_1 \mathrm{T},$$

or
$$\log_\epsilon y = \frac{m_1}{2\beta}\log_\epsilon \frac{m_2}{m_1},$$

$$\therefore \quad y = \left(\frac{m_2}{m_1}\right)^{\frac{m_1}{2\beta}}.$$

Again,
$$\theta_{\max} = \frac{\mathrm{M}}{\mathrm{G}}x, \quad x = \frac{-2a}{m_2}\left(\frac{m_2}{m_1}\right)^{\frac{m_1}{2\beta}}.$$

Fluxmeter.—

$$x = \frac{-2a}{-a-\beta}\left(\frac{-a-\beta}{-a+\beta}\right)^{\frac{-a+\beta}{2\beta}},$$

$$x = \frac{2a}{a+\beta}\left(\frac{a+\beta}{a-\beta}\right)^{\frac{-a+\beta}{2\beta}}.$$

This is a form more amenable to arithmetical computation than the former expressions.

Value of G.—From data supplied by the manufacturers, it appears that M = 1020 per division,

or
$$\frac{1 \cdot 02 \times 57 \cdot 3 \times 10^3}{\cdot 4} \text{ per radian,}$$

$1 \cdot 461 \times 10^6$ line turns per radian.

But
$$\mathrm{M} = \mathrm{G} \,.\, \theta_{\max},$$

$$\mathrm{G} = 1 \cdot 46 \times 10^6.$$

Value of k.—The instrument took 30 micro-amperes per 50 divisions and since $Gc = k\theta$, we have

$$1{\cdot}46 \times 10^6 \times 30 \times 10^{-7} = k\frac{50}{57{\cdot}3} \times {\cdot}4 \text{ (see page 51)},$$

$$k = 12{\cdot}55,$$

dyne centimetres per radian.

Checking Results.—Let

$$\beta = 0{\cdot}95a,$$

and
$$x = \frac{2a}{a+\beta}\left(\frac{a+\beta}{a-\beta}\right)^{\frac{-a+\beta}{2\beta}}.$$

$$\therefore \quad x = \frac{2}{1{\cdot}95}{\cdot}39^{-1/38}$$

$$x = 0{\cdot}931.$$

Now the deflection is

$$125 \times 0{\cdot}931,$$

$$\therefore \quad \theta_{\max} = 116{\cdot}4.$$

Again
$$\beta^2 = a^2 - b,$$

$$-\beta^2 + a^2 = b,$$

$$-{\cdot}95a^2 + a^2 = b.$$

$$a^2 = \frac{b}{0{\cdot}1},$$

$$a = 3{\cdot}2\sqrt{b},$$

From the curve below it is seen that the resistance in the circuit when the deflection is 116·4 is 102 ohms, and the agreement generally is very fair.

In the above way the following table was made up :

β	a	x	Deflection.	R total.
$0{\cdot}8a$	1·66	·844	105·5	195
$\cdot 9a$	2·395	·894	111·8	135
$\cdot 95a$	3·16	·931	116·4	102
$\cdot 975a$	4·47	·958	119·8	72
$\cdot 99a$	7·07	·979	122·4	45·5

It will be noticed that when $a = 3{\cdot}16\sqrt{b}$, R $= 102$ from curve, and when $a = 1{\cdot}66\sqrt{b}$, R $= \dfrac{102 \times 3{\cdot}16}{1{\cdot}66} = 195$, since

$$a \propto \frac{1}{\text{R}}.$$

The results of experiment and calculation are exhibited in the following curve (Figs. 7 and 8).

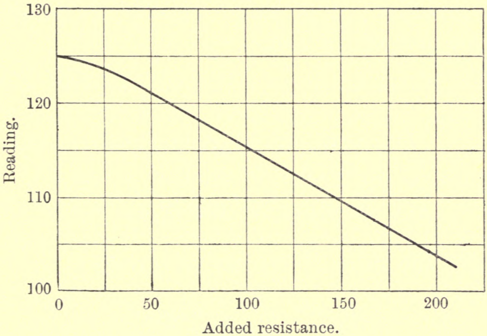

Fig. 7.—Test on Grassot Fluxmeter.

The periodic time T was extremely difficult to determine accurately, as the control is very weak, and when

on open circuit, the movement is not symmetrical on both sides. When moving in one direction the ligaments

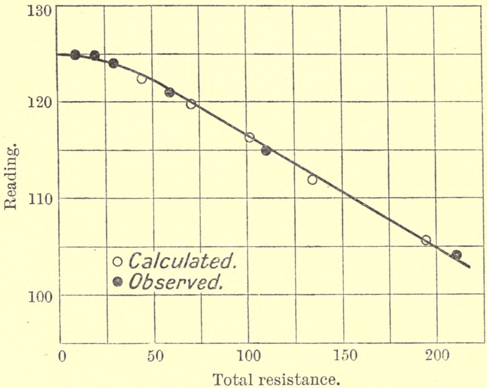

FIG. 8.—Test on Grassot Fluxmeter.

appear to tighten, and loosen again when moving in the opposite direction, and these appear to produce a variable amount of friction.

DAMPING OF MOVING COIL

Since $$\theta = \theta_{\max}\epsilon^{-at/2} \cos \beta t,$$

we have for the ratio of successive throws :

$$\frac{\theta_1}{\theta_{n+1}} = \frac{\theta_{\max}\epsilon^{-at_1/2}}{\theta_{\max}\epsilon^{-at(n+1)/2}},$$

$$\therefore \quad \frac{\theta_1}{\theta_{n+1}} = \epsilon^{a\mathrm{T}n/4}.$$

With reference to the open circuit test, Table I., p. 54, the eleventh throw is 44, the first 90, therefore

$$\left(\frac{44}{90}\right)^{\frac{1}{10}} = 0 \cdot 9309\,;$$

this refers of course to coil 2.

Multiplying the throws by the corresponding ratio gives a very close approximation to the observed throws, as is seen from the Table I., p. 54.

Determination of a.—Since

$$T = \frac{2\pi}{\beta} = 5 \cdot 22 \text{ secs.},$$

$$\epsilon^{-aT/2} = (0 \cdot 9309)^2,$$

$$\therefore \quad aT = \frac{2 \times 0 \cdot 06214}{0 \cdot 4343},$$

$$\therefore \quad a = 0 \cdot 0565.$$

Since in this case the coil is on open circuit, the damping factor a is due solely to windage, since no current can circulate in the galvanometer coil.

If now coil 1 is short circuited, the damping will be due to both windage and induced currents.

From the Table, Coil 1, p. 56, we have

$$\log\left(\frac{10}{90}\right)^{\frac{1}{8}} = -1 \cdot 8807.$$

This makes a_A the damping factor $0 \cdot 308$; if we deduct the previous value $0 \cdot 0566$ obtained on open circuit (assuming air damping the same as previously, which

can hardly be the case since the coil is not now oscillating), we obtain

$$a_{\text{A}} - a = 0{\cdot}2514,$$

due to induced currents.

Writing

$$\theta = \text{A}\epsilon^{m_1 t} + \text{B}\epsilon^{m_2 t},$$

or
$$\theta = \frac{\theta_{\max}}{-m_1 + m_2}\left\{-m_2\epsilon^{m_1 t} + m_1\epsilon^{m_2 t}\right\},$$

$$m_2 = -\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}, \quad \frac{a^2}{4} - b \text{ is } +\text{ve},$$

$$\therefore \quad a \text{ must be } > 1{\cdot}2.$$

When t is as low as $2{\cdot}5$, $\epsilon^{m_2 t}$ must be less than $\frac{1}{\epsilon^{1\cdot 2\times 2\cdot 5}}$, *i.e.* $< \frac{1}{\epsilon^3}$.

Hence $\text{B}\epsilon^{mt}$ must be negligibly small. The quantity $\frac{-m_2}{-m_1 + m_2}$ is also nearly equal to unity, so that

$$\theta = \theta_{\max}\epsilon^{m_1 t}\frac{m_2}{m_2 - m_1} = \theta_{\max}\epsilon^{m_1 t} \text{ approximately}$$

for the short circuit reading at any time t.

The readings may be calculated from the decrement observed above and compared with the observed readings as previously. This is shown in the following table :

[TABLE

Logarithms.	Calculated.	Observed from Curve.
·9542	90	90
880725		
834925	68·37	68
71565	51·96	53
596375	39·40	41
477100	30·00	30
357825	22·79	24
238550	17·32	18
119275	13·16	13
000000	10·00	10

Again, when $T=10$ seconds the reading was 30 divisions

$$\epsilon^{m_1 T}=\frac{1}{3},$$

$$\therefore \quad mT \times \cdot 4343 = -\cdot 4747,$$

or

$$m_1 = 0\cdot 1085,$$

and $a=\dfrac{b}{m_1}$ approximately.

$$a=\frac{1\cdot 48}{0\cdot 1085}=13\cdot 65,$$

$$13\cdot 65 - \cdot 057 = 13\cdot 59.$$

Hence G^2/I.

Short circuit coil (1)

$$\overset{\text{Resistance}}{13{\cdot}59 \times 36{\cdot}5 = 496.}$$

Coil (1) $\qquad 0{\cdot}2514 \times 2036 = 512.$

Again, with coil 1, total resistance 2035·4 ohms, the swings were 90, 60, 40, 27, 18, 11, 8,

$$\log\left(\frac{18}{90}\right)^{\frac{1}{4}} = \tfrac{1}{4}\log\frac{1}{5},$$

$$= -{\cdot}82525.$$

Calculation of Periodic Time.—

$$-\frac{aT}{2} \times 0{\cdot}4343 = 2(-{\cdot}82525),$$

$$\therefore \quad a = 0{\cdot}308;$$

from the open circuit test

$$T = \frac{2\pi}{\sqrt{b}},$$

$$b = \left(\frac{2\pi}{T}\right)^2 = \left(\frac{6{\cdot}28}{5{\cdot}06}\right)^2 = 1.48.$$

T in test therefore is

$$T = \frac{2\pi}{\sqrt{-\left(\frac{0{\cdot}154}{I}\right)^2 + 1{\cdot}48}},$$

$$T = \frac{2\pi}{\sqrt{1{\cdot}48 - 0{\cdot}237}},$$

$$\therefore \quad T = 5{\cdot}2.$$

The actual T was 5·22, so that the agreement is very exact.

Effect of Self-Induction on Damping.—If L is the self-induction coefficient of a transformer coil, we may write

$$L = \frac{E}{2\pi n C}.$$

Let $E = 100$ volts, $n = 40$, $C = 0{\cdot}3$ with open secondary, then

$$L = 1{\cdot}3 \text{ Henries.}$$

It appears that inserting this value of self-induction (the above only to indicate the order of magnitude) in circuit is practically without any effect.

With $R = 1000$, 500, 11, 1, and 0·1 ohm, there was practically no difference in the damping with the coil in circuit, or out of circuit.

On p. 47 we omitted to take into account the term

$$cL\frac{dC}{dt}$$

in equation of motion, and this appears to be justified in this case.

Value of I *for Grassot Fluxmeter.*—The moment of inertia of the moving portion may be found as follows :

Let $R = 102$ ohms.

$$a = 3{\cdot}16\sqrt{b},$$

$$\frac{G^2}{RI} = a = 2a = 6{\cdot}32\sqrt{b},$$

$$\sqrt{b}I = \frac{G^2}{R \,.\, 6{\cdot}32},$$

$$\sqrt{k/\mathrm{I}} \times \mathrm{I} = 33{\cdot}2,$$

$$\therefore \quad \sqrt{\mathrm{I}} = \frac{33{\cdot}2}{\sqrt{k}},$$

$$\therefore \quad \mathrm{I} = \frac{33{\cdot}2}{12{\cdot}55},$$

$$= 88 \text{ grm. cm.}^2,$$

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{I}}{k}} = 2\pi\sqrt{\frac{88}{12{\cdot}55}},$$

$$\therefore \quad \mathrm{T} = 16{\cdot}7 \text{ seconds,}$$

$$\text{observed } \mathrm{T} = 15 \text{ seconds.}$$

Logarithmic Decrement

As a general rule, the damping of the motion of moving coils, needles or pendulums, depends on two factors. These are the resistance of the surface to the motion, and the increased inertia due to the moving body carrying air with it.

The mathematical aspect of the question has been considered first by Sir G. G. Stokes, "On the Effect of the Internal Friction of Fluids on the Motion of Pendulums" (*Scientific Papers*, vol. iii. p. 1), and by Basset "On the Motion of a Sphere in a Viscous Liquid" (*Hydrodynamics*, chap. xxii.). The results of these calculations show that the resultant force on a moving body is generally of the type

$$\mathrm{F} = \mathrm{V}\left(\frac{1}{2} + \frac{1}{\mathrm{K}}\right)\frac{dv}{dt} + kv.$$

Where V is the volume of the moving body, $\frac{1}{2}$ is the increase in inertia for a frictionless liquid, and $\frac{1}{K}$ is the increased inertia for viscosity, and $\frac{dv}{dt}$ the acceleration. The latter term kv is simply the force resisting motion varying with the velocity. Now for most moving bodies considered in this book the former term is negligible, and the resisting force is

$$F = -K\frac{d\theta}{dt},$$

as already taken into account in the differential equation above considered, viz.

$$\frac{d^2\theta}{dt^2} + a\frac{d\theta}{dt} + b\theta = 0$$

for a damped oscillation.

The general effect of damping is to diminish the oscillations in a fixed proportion. It does not appreciably alter the periodic time, although it increases it by a small quantity of the second order, as shown above, p. 61.

Since the energy of a vibration depends on the potential energy stored up at the end of a swing, the energy in moving through a small space $d\theta$ is ($a\theta\, d\theta$), and the whole energy $a\int\theta d\theta$ or

$$\frac{b\theta^2}{2},$$

the angle being measured from θ to 0.

Similarly the work done by friction is $\frac{a\theta^2}{2}$, so that the energy left for the following swing is

$$\frac{b-a}{2}\theta^2.$$

This applies to any swing, and if we write $\frac{b-a}{2}=\frac{1}{\rho^2}$ we see that it corresponds to an amplitude diminishing in the ratio $\frac{1}{\rho}$.

Hence if θ_1 is the first swing and θ_n the nth swing,

then $$\theta_n=\frac{\theta_1}{\rho n-1}.$$

If now we write $$\lambda=\log_\epsilon \rho,$$

we have $$(n-1)\log_\epsilon \rho=\log_\epsilon \theta_1-\log_\epsilon \theta_n$$

or $$\lambda=\frac{1}{n-1}\left\{\log_\epsilon \theta_1-\log_\epsilon \theta\right\}.$$

To correct any swing (a quarter oscillation) we have

$$\lambda=\frac{1}{\frac{1}{4}}(\log_\epsilon \theta_0-\log_\epsilon \theta)$$

or $$\tfrac{1}{4}\lambda+\log_\epsilon \theta=\log_\epsilon \theta_0,$$

where θ_0 is the corrected swing.

But $$\epsilon^{\log_\epsilon \theta_0}=\epsilon^{\frac{1}{4}\lambda+\log_\epsilon \theta},$$

or $$\theta_0=\theta\,.\,\epsilon^{\frac{1}{4}\lambda},$$

$$\theta_0=(1+\tfrac{1}{4}\lambda)\theta$$

approximately by logarithmic theorem.

Best Value for Ratio in finding the Logarithmic Decrement.—

Let R be the ratio of the swings, m the number of oscillations, then

$$\lambda = \frac{1}{m-1} \log_\epsilon \mathrm{R},$$

$$\frac{d\lambda}{\lambda} = \frac{d\mathrm{R}}{\mathrm{R} \log_\epsilon \mathrm{R}}.$$

In order that $\frac{d\lambda}{\lambda}$ is to be a minimum, $\mathrm{R} \log_\epsilon \mathrm{R}$ must be a maximum. Hence $\mathrm{R} \log_\epsilon \mathrm{R}$ is to be a maximum.

Let $$y = \mathrm{R} \log_\epsilon \mathrm{R},$$

$$\frac{dy}{d\mathrm{R}} = -\log_\epsilon \mathrm{R} + 1 = 0,$$

$$\therefore \quad \mathrm{R} = \epsilon^1.$$

Consequently in order to reduce errors to a minimum, the ratio of swings should approximate to 2·718.

In this case the error will then be

$$\frac{d\lambda}{\lambda} = \frac{d\mathrm{R}}{\epsilon}.$$

In other words if $d\mathrm{R} = \frac{1}{100}$ the percentage error is $\frac{1}{\epsilon}$ or 0·43 per cent.

Eddies in Moving Coil Instruments

In this case, if we assume a magnetic flux acting on a coil, as shown in section on Damping (Fig. 9), it is easily shown that the mean torque

Similarly the work done by friction is $\frac{a\theta^2}{2}$, so that the energy left for the following swing is

$$\frac{b-a}{2}\theta^2.$$

This applies to any swing, and if we write $\frac{b-a}{2}=\frac{1}{\rho^2}$ we see that it corresponds to an amplitude diminishing in the ratio $\frac{1}{\rho}$.

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If now we write $$\lambda=\log_\epsilon \rho,$$

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To correct any swing (a quarter oscillation) we have

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or $$\tfrac{1}{4}\lambda+\log_\epsilon \theta=\log_\epsilon \theta_0,$$

where θ_0 is the corrected swing.

But $$\epsilon^{\log_\epsilon \theta_0}=\epsilon^{\frac{1}{4}\lambda+\log_\epsilon \theta},$$

or $$\theta_0=\theta\,.\,\epsilon^{\frac{1}{4}\lambda},$$

$$\theta_0=(1+\tfrac{1}{4}\lambda)\theta$$

approximately by logarithmic theorem.

Best Value for Ratio in finding the Logarithmic Decrement.—

Let R be the ratio of the swings, m the number of oscillations, then

$$\lambda = \frac{1}{m-1}\log_\epsilon R,$$

$$\frac{d\lambda}{\lambda} = \frac{dR}{R\log_\epsilon R}.$$

In order that $\frac{d\lambda}{\lambda}$ is to be a minimum, $R\log_\epsilon R$ must be a maximum. Hence $R\log_\epsilon R$ is to be a maximum.

Let $$y = R\log_\epsilon R,$$

$$\frac{dy}{dR} = -\log_\epsilon R + 1 = 0,$$

$$\therefore \quad R = \epsilon^1.$$

Consequently in order to reduce errors to a minimum, the ratio of swings should approximate to 2·718.

In this case the error will then be

$$\frac{d\lambda}{\lambda} = \frac{dR}{\epsilon}.$$

In other words if $dR = \frac{1}{100}$ the percentage error is $\frac{1}{\epsilon}$ or 0·43 per cent.

Eddies in Moving Coil Instruments

In this case, if we assume a magnetic flux acting on a coil, as shown in section on Damping (Fig. 9), it is easily shown that the mean torque

$$T = \dagger\left(\frac{BAN}{10^9}\right)^2 \frac{\pi n \sin\phi}{\sqrt{R^2 + (2\pi n L)^2}},$$ See errata

where ϕ is angle of lag, n is periodicity, L is self-induction coefficient.

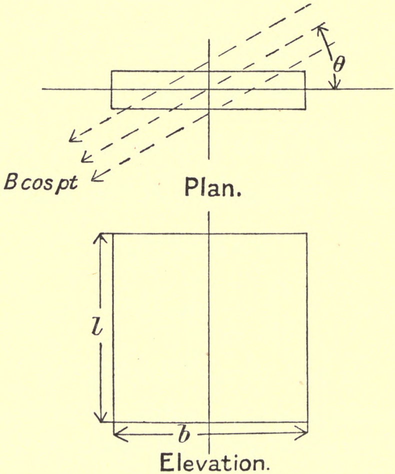


Fig. 9.

Consequently when $\phi = 0$, $\sin\phi = 0$, and torque is zero. Assuming such to be the case, the torque will

$$\propto \frac{\sin\phi}{\sqrt{R^2 + (2\pi n L)^2}},$$

or
$$T \propto \frac{2\pi n L}{R^2 + (2\pi n L)^2},$$

and if n and L are constant, putting $2\pi n L = x$,

$$T(R^2 + x^2) = k.$$

Experiment.—To test this a coil similar to a voltmeter coil, with the exception that it was wound on an ebonite former, was used, length 1·5 cms., breadth $=1·5$ cms., $N=200$ turns, $R=92$ ohms, the wire being No. 47 S.W.G. double silk-covered copper. This coil was placed between the poles of an alternating current magnet A.C., volts 86, $n=38$. The distance of scale from the mirror attached to the coil was 73·7 cms.

(*a*) When the coil was short circuited no deflection was obtained.

(*b*) With 100 volt coil of 100 : 50 ratio transformer in series and different added resistances, the following table of results was obtained :

Added Resistance.	Deflection in Cms.	Added Resistance.	Deflection in Cms.
0	12·4	80	5·05
10	11·2	100	4·25
20	9·8	150	2·8
40	7·9	200	2·15
60	5·9		

When the deflection was 2·8, $R_1=150+92=242$.

When the deflection was 5·05, $R_2=80+92=172$.

From these results $x=6·4\times10^3$, using above equation, and $k=182000$.

Calculating as a check for R, 100, 200, 300 respectively, we obtain 11·1, 3·94, and 1·90, which agrees very fairly with the second, seventh and last observation in above table.

Strictly these results require correction for angle turned through by the coil, but these corrections are small.

Similar results were obtained with larger and smaller self-inductions and resistances in series.

Transformer Data.—Turns in HT, 248.
Turns in LT, 128.

Resistance of these, 0·8 ohm and 0·2 ohm respectively.

Data re Coil.—The coil was placed in a field of strength 900 lines per square centimetre (by Grassot flux-meter), and the deflection was 9·8 cms. at a distance of 73·7 cms. when a current of 5×10^{-4} ampere was passed through it.

A coil fitted with a copper damping ring was tried, and large deflections obtained which appear to show that the reactance compared with the resistance of the ring may be appreciable in this case.

Calibration of Grassot's Fluxmeter and Ballistic Galvanometer for Magnetic Work

Let r = total resistance on the circuit.
E = E.M.F. induced on exploring coil at any instant.
I = instantaneous current.
ω = angular velocity.
θ = angular displacement of coil.
L = coefficient of self-induction of the whole circuit.
Mk^2 = moment of inertia of moving coil.
$-K\omega$ = E.M.F. induced on moving coil owing to its movement.
CI = turning moment on coil at any instant.
$-A\omega$ = the turning moment due to air resistance.

Hence since L, K, C, A, are all constants,

$$E - Ir - K\omega - L\frac{dI}{dt} = 0 \quad . \quad . \quad \text{(i.)}$$

$$Mk^2\frac{d\omega}{dt} = -A\omega + CI \quad . \quad . \quad \text{(ii.)}$$

From (i.)

$$I = \frac{E - K\omega - L\frac{dI}{dt}}{r} \quad . \quad . \quad \text{(iii.)}$$

Substituting in (ii.) gives

$$Mk^2\frac{d\omega}{dt} = -\left(A + \frac{CK}{r}\right)\omega + \frac{C}{r}E - \frac{CL}{r}\,\frac{dI}{dt}.$$

In both initial and final positions of the coil $\omega = 0$, $I = 0$,

$\therefore \quad \frac{CL}{r}\,\frac{dI}{dt}$ and $Mk^2\frac{d\omega}{dt}$ may be omitted.

$$0 = -\left(A + \frac{CK}{r}\right)\int \omega dt + \frac{C}{r}\int Edt,$$

$$\therefore \quad \int Edt = \left(\frac{A}{C}r + K\right)\theta.$$

Since A, C, r, and K, are all constants, we have the result that the final deflection θ is due to the E.M.F. impulse

$$\int Edt = k\theta,$$

where k is some constant.

But

$$E = \frac{dN}{dt},$$

$$\therefore \quad \frac{dN}{dt}dt = k\theta,$$

$$N_1 - N_2 = k\theta,$$

where $N_1 - N_2$ is the change of flux. To standardise, then, the search coil may be inserted in a standard field and the divisions determined in Maxwells.

Since $$\int E dt = k\theta$$

and $E = C \times R$ we see that

$$\int C dt \times R = k\theta.$$

Hence the instrument could be calibrated by means of discharge of a condenser.

$$\int C dt = \frac{k\theta}{R} \cdot$$

For this purpose, the instrument is arranged thus :

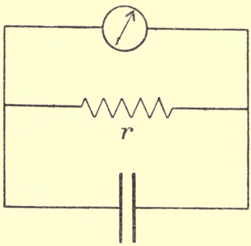


FIG. 10.—Calibration of Grassot's Fluxmeter.

If M are the Maxwells per division, then

$$Q = \frac{M\theta}{r \times 10^8}$$

in coulombs.

Ballistic Galvanometer.—Can be calibrated by :

1. Discharge of condenser.
2. Standard solenoid.
3. Direct deflection.

In (i.) a condenser is charged to a known potential and then discharged through the galvanometer. Neglecting damping, as we have already seen (p. 48),

$$Q = \frac{H}{G} \cdot \frac{T}{\pi} \sin \tfrac{1}{2}\theta.$$

But

$$Q = \int C dt = \int \frac{E}{R} dt,$$

$$E = \frac{dN}{dt},$$

and

$$Q = \frac{\int \frac{dN}{dt} \cdot dt}{R}.$$

If R, therefore, is the total resistance in circuit,

$$\int_{N_1}^{N_2} dN = R \cdot \frac{H}{G} \cdot \frac{T}{\pi} \sin \tfrac{1}{2}\theta,$$

where H refers to moving magnet K in case of moving coil.

Also $V_1 = 92\sqrt{V_2}$ approximately.

This will be noticed from the following table of results :

V_1.	V_2.	$V_1 = 92\sqrt{V_2}$.
29	0·1	29
39	·2	41
51	·3	50·3
59	·4	58
66	·5	65
71	·6	71

Hence the reading is proportional to the square of the alternating voltage across the instrument.

The zero was then shifted by altering spring control, as only one spring was used, flexible leads being used to carry the current in this instrument, and the following readings were obtained :

Initial Reading.	Plus Alternating Current.	Deflection due to Alternating Current.
0	0·5	+ 0·5
2	2·3	0·3
3·8	4·0	0·2
5·7	5·75	0·5
7·8	7·7	− 0·1
9·8	9·55	− 0·25
11·75	11·30	− 0·45

The divisions were practically equal on the scale. Length of pointer $3\frac{15}{16}$ inches, and chord between 0

and 12 reading was $4\frac{15}{16}$ inches. The coil was at right angles to the pointer.

We see from the test that when the axis of the coil is at right angles to the line joining the magnet poles, no effect is produced, and that the effect increases as the coil becomes more nearly parallel to the line joining the poles. Hence it would appear that the coil produces a flux which completes its path through the magnet poles of the instrument.

In considering the cause of these results, since the effect is not dependent on periodicity, it would seem that it is due to the flux being increased or diminished as the current through the moving coil changes its direction.

Hence for the steady deflection observed we have, considering maximum values only, since

$$H_1 - H_2 = a \cdot C_m,$$

where $H_1 - H_2$ is flux density change, a is a constant, Cm is maximum current, then torque

$$T \propto (H_1 - H_2) C_m$$

or $$T \propto C_m \{(H + aC_m) - (H - aC_m)\},$$

that is $$T \propto 2aC_m^2.$$

And since $T = k\theta$ we see that $\theta \propto C_m^2$, or generally $\theta \propto C^2$ where C is the R.M.S. current. $H \pm aC_m$ is flux density with alternating current.

In ordinary instruments the field is a radial one, thus :

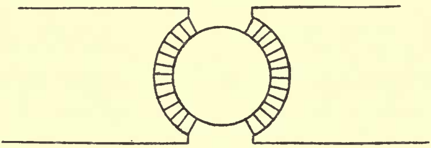


FIG. 12.—Air gap flux of moving coil instrument.

and the maximum movement of the coil is fixed by field uniformity, so that the angle of deflection is usually about 80° to 90°.

The difficulty of obtaining a larger angle has recently been ingeniously overcome by Mr. J. W. Record, A.M.I.E.E., who used a magnet shaped as shown in Fig. 13, the field being arranged perpendicular to the central disc.

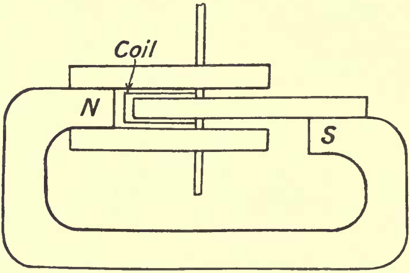


FIG. 13.—Record instrument.

In consequence of this arrangement, the arc is now increased to 300° or, if necessary, to 330°, and the scale divisions are equal throughout the range, and the gap resistance is reduced.

Double controlling springs are generally used wound in opposite directions to get rid of the expansion error due to changes in temperature in all good permanent magnet instruments. In some instruments the current is led in by silver strip producing no control, and in this case only *one* spring is used.

Magnets for use in such instruments should be long and the air gap should be small in order to ensure permanence. The stronger the field the greater the deflection for a given current and *vice versa*.

The errors to which such instruments are subject are:

1. Temperature error affecting spring, if only one.

2. Temperature causing a change in field strength of the magnet.

3. Temperature altering rigidity of spring.

4. Ageing of magnet.

As a rule the loss of magnetism for rising temperature is greater than the gain on cooling, so that changes of temperature may cause a permanent change of magnetism.

As an approximation we might write M_0 = moment of magnet at temperature θ_0, M_t = moment at θ, then

$$M_t = M_0\{1 - 0{\cdot}0035(\theta - \theta_0)\}$$

between 50° and 100° C. (*vide* Stewart and Gee, vol. i. p. 45).

We see, therefore, that for 10° C. rise of temperature there is a change of about 3 per cent in the magnetic moment. In Ewing's *Magnetic Induction in Iron and Other Metals* will be found many experiments on the effect of temperature (p. 183). The effect shown in Ewing for an oil hardened steel magnet for a temperature range of 10° to 158° C. shows only a very small effect and no hysteresis relative to temperature. The effect for lower inductions is greater than for high, so that strongly magnetised steel is essential.

Theory of the "Megger"

The magnetic circuit of the ohmmeter portion of this most ingenious instrument is illustrated in Fig. 14.

The soft iron core consists of a hollow cylinder with a gap as shown. It is clear that the flux in the gap will be more intense on that side in which the current coil moves. Also it will be noticed that the pole near A is cut away, forming a neck, over which the compensating coil of the potential system is capable of moving.

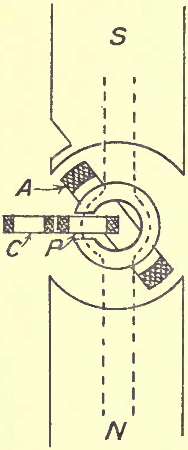


FIG. 14.

P, Pressure coil.
A, Current coil.
C, Compensating coil.

In Fig. 15 the arrangement of the actual instrument is shown. The scale divisions of this instrument are by no means uniform; consequently the law of the instrument is apparently not a simple linear one, neither is it a tangent or other simple function.

The arrangement of the moving system is shown in Fig. 16.

The greater the current passing through the current coil the greater the deflection, and it is apparent that variation of voltage (see Fig. 16) will affect both potential and current coils equally; consequently it will have no effect on the reading.

Assume then that the current coil moves in a gap of constant flux density, and assume that the flux cut by the potential coil which embraces the iron core, and also the flux cut by the compensating coil, varies as some function of θ, then we have for the current coil, flux radial, and constant, torque

$$=k_2 i.$$

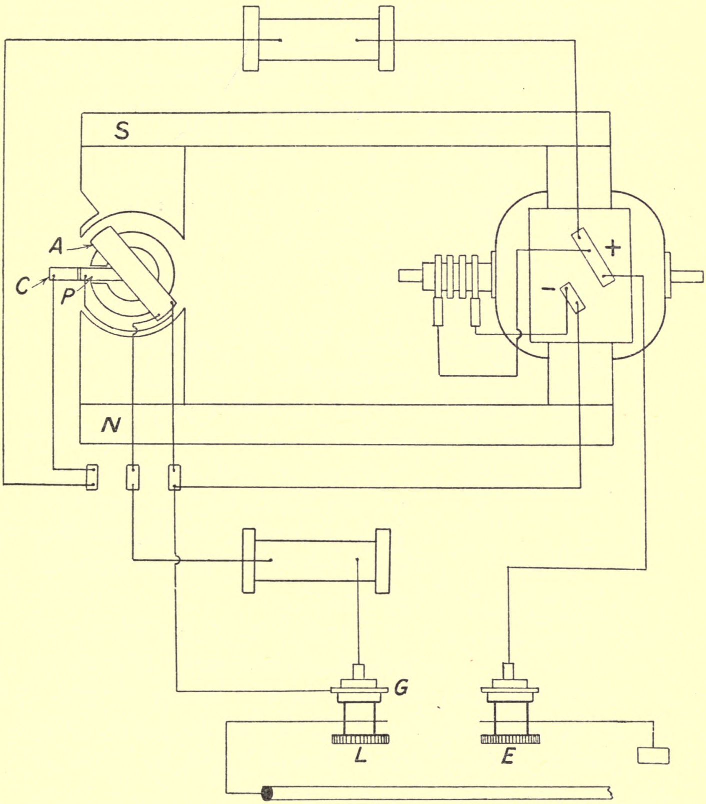


FIG. 15.—The Megger.

A, Current coil.
P, Pressure coil.
C, Compensating coil.
N, S, Magnets.

+ −, Generator terminals.
G, Guard plate.
L, E, External (line and earth) terminals.

For the potential coil, torque $= k_1 v f(\theta)$.

For equilibrium we must have

$$k_1 v f(\theta) = k_2 i.$$

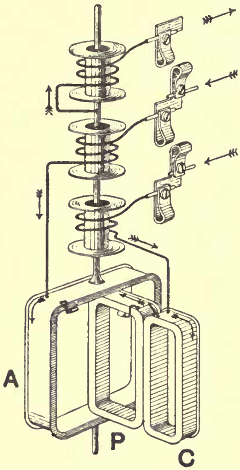


Fig. 16.—Coils of the Megger.

P, Pressure coil.
A, Current coil.
C, Compensating coil.

Hence $\frac{v}{i} \propto \frac{1}{f(\theta)}$.

But $\frac{v}{i} = \mathrm{R}$.

Hence $\mathrm{R} \propto \frac{1}{f(\theta)}$,

where R is the resistance being measured.

The generator used in conjunction with the measuring mechanism and forming part of the instrument is arranged with a friction clutch, so that the voltage is independent of speed.

By means of an additional resistance box, another form of the instrument can be used as a "Bridge Megger" measuring resistances over a very wide range. The connections of the "Bridge Megger" will be found in Fig. 17.

There are several alternative methods of using the instrument, and for continuous testing in the laboratory it may conveniently be driven by means of a small motor.

CAPACITY EFFECT IN "MEGGER"

If we regard the E.M.F. generated by the generator in this instrument as of the form

$$e = \mathrm{E} + e_1 \sin pt +, \text{ etc.},$$

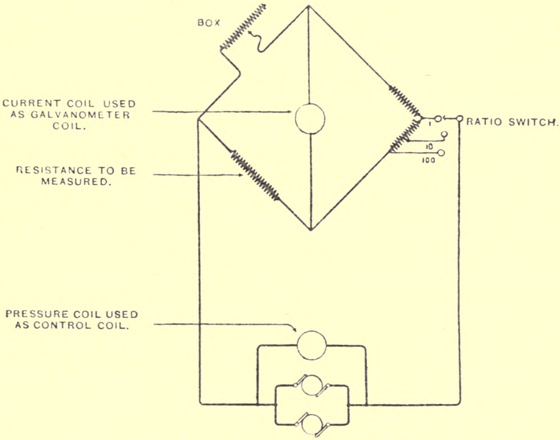


FIG. 17.—The Bridge Megger.

the capacity current will be

$$\mathrm{I} = \frac{e_1 \sin (pt + \phi)}{\mathrm{Z}_1} + \frac{e_3 \mathrm{Z}_3 \sin (3pt + \phi_3)}{\mathrm{Z}_3} +, \text{ etc.},$$

where Z_1 and Z_3 is the impedance.

The torque due to this will, if the field strength be constant,

$$\propto \int_0^{\mathrm{T}} \mathrm{I} dt$$

or $$\propto \int_0^T \frac{e_1 \sin(pt+\phi)}{Z_1} dt +, \text{ etc.},$$

which is clearly zero.

On testing with 100 microfarads in circuit, no capacity effect could be observed.

On first rotating handle, and whenever speed varies, the D.C. capacity charging current produces a large throw. Hence constant speed required.

On the other hand, in the earlier type of this instrument, the ohmmeter, quite an appreciable capacity effect is observed even when it is running at a constant speed.

The Ducter

Consider a permanent magnet NS, Fig. 18, with hollow cylindrical piece of soft iron between its poles. The flux into the ring is assumed radial, so that for a moving coil entirely surrounding the cove the flux is uniform for any angular position. In the cove, however, we see that the flux at a, say, is a maximum diminishing to zero at b. If, therefore, we have two coils A and B, A arranged as an ordinary D'Arsonval coil, B arranged to embrace the flux in the core, then

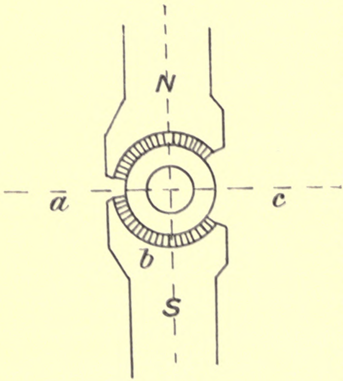


Fig. 18.

the following occurs: If current is passed through B alone, it will move to *a*, if current is passed through A alone it will move to *c*, or lie in the diameter *ac*.

If current now is arranged to pass through A in such a way as to make it move in the direction of the arrow and B to move in the opposite direction, it is clear that for any values of current the coil will take up certain definite positions. Let Z be the flux cut by the coil A, let $Z'\theta$ be the flux embraced by B in the core at any point, $n_1 n_2$ the turns on the coils, the radii being r and $2r$. For equilibrium we must have

$$rn_2 Z'\theta i = 2rZken_1$$

where i is the current proportional to the main current and e is the current proportional to volt drop and k is a constant.

Hence $$\theta = \frac{2Zkn_1}{Z'n_2} \frac{e}{i}.$$

But Z is constant, Z' the maximum flux in the hollow cylinder is also constant, n_1 and n_2 are constant,

$$\therefore \quad \theta = K\frac{e}{i} \text{ where } K = \text{constant.}$$

Hence $$\theta \propto R.$$

Hence by having two coils, both wound potentially, for taking readings, it is possible to design an instrument which will indicate resistance over a considerable range.

It is apparent that the coil B must be adjusted to move through some angle much less than 90°, since the readings depend on uniformity of variation of flux for equal angular displacements.

In Evershed's Ducter the general arrangement is that shown in Figs. 19, 20, 21. The angular motion appears to be about 60°, and the arrangement of resistance shown enables the instrument to give various grades of sensitiveness.

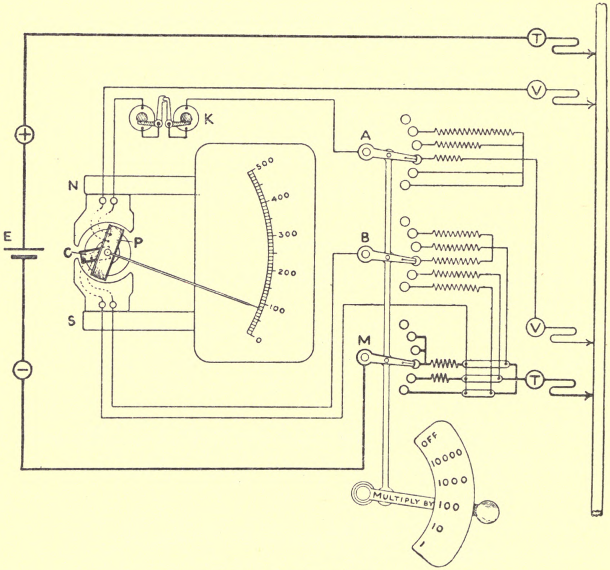


FIG. 19.—The Ducter.

E, Battery.
C, Current coil.
P, Potential coil.
N, S, Poles of magnet.
K, Cut-out.
A, B, M, Triple-pole grade switch.
X, Resistance under test.
H, G, Potential contacts.

Both the leads T and V are combined in a handle with two spikes, so that when any one wishes to measure the resistance between any two points he takes a handle in each hand and presses them on the points

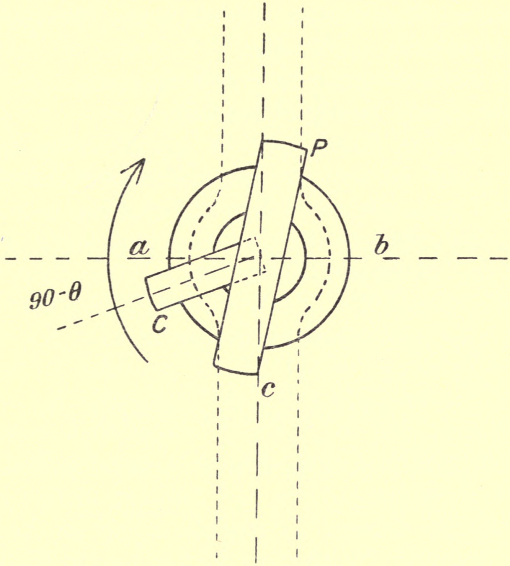


FIG. 20.—The Ducter moving system.

P, Potential coil. C, Current coil.

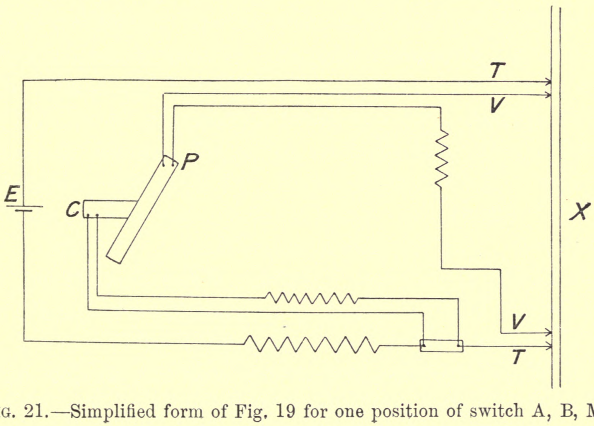


FIG. 21.—Simplified form of Fig. 19 for one position of switch A, B, M.

between which the resistance is required. The resistance is then indicated on the dial.

It is needless to say that such an instrument can be used for a great variety of purposes.

For making direct tests of specific resistance the arrangement of current and potential terminals are mounted upon a wooden base and capable of being set at any given distance apart, so that the specific resistance can be read direct from the ducter.

This instrument, together with the "Megger," is one of the most ingenious in use at present.

Vibration Galvanometers

These have been devised by Campbell and Duddell.

In the Duddell pattern, the damping of the moving wires is made as small as possible, and it can be timed to any frequency by altering the tension and the length of the vibrating wires by means of a milled headed screw at top and bottom of the wires.

In Campbell's type, a very narrow coil moves in the air gap of a permanent magnet. It is suspended bifilarly, the tension of which can be adjusted by means of a spiral spring. For the lower frequencies 50 to 200, all one has to do is to turn the milled head outside the case, so that it is very easily tuned. To tune the instrument a small current is passed through the coil at steady frequency, and the milled head turned until the light spot broadens out into a band. When this attains its maximum length, tuning is complete. The length of band measures the current passing.

It is a resonance instrument working on the same principles as those of Rubens and Professor Max Wien.

These instruments are particularly useful for measuring small inductances and capacities, as also electrolytic resistance. They are suitable for use in null methods of testing, such as Maxwell's, Wien's, Carey Foster's, or Anderson's.

When testing very small inductances, the galvanometer is connected to a small transformer.

A steady source of current is to be preferred, but the galvanometer works well with tuning fork interrupter giving ranges of frequency of 600 to 700 per second.

The following illustrates the use of the instrument; for further information the reader must consult an article by A. Campbell (*Phil. Mag.*, Jan. 1908).

Mutual Inductance.—Let P_1S_1 be the primary and secondary winding of a standard mutual inductance,

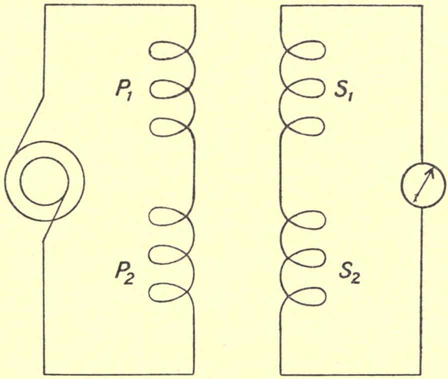


FIG. 22.

P_2S_2 the mutual inductance to be tested. The secondaries being connected in opposition, the variable inductance is then adjusted to bring the deflection to

zero, and the reading of the variable inductance gives the value of the unknown.

Measurement of Self-Inductance.—RR are equal non-inductive arms. The secondary of the standard is inserted in AB; in the arm AC, a balancing coil L.

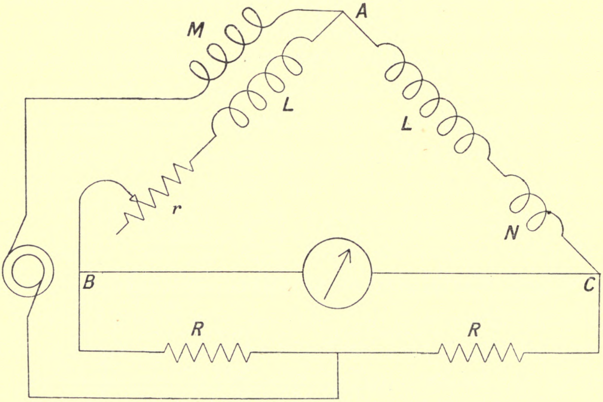


FIG. 23.

Adjust rheostat r till the bridge balances. Then introduce N and balance by altering r and M. Then

$$N = 2M.$$

Consequently you can by this method test up to twice the value of the variable standard. Unequal arms will enable tests for higher ranges to be made.

Capacity.—For larger capacities the primary P of the variable standard is connected in series with a condenser to a source of periodic current, while the secondary Q is connected to the vibration galvanometer. M is adjusted till no deflection takes place.

Then $p^2MK = 1$, where $p = 2\pi n$. Hence K is obtained.

For small capacities the method of Carey Foster may be used (see *Phil. Mag.* p. 424, Oct. 1907).

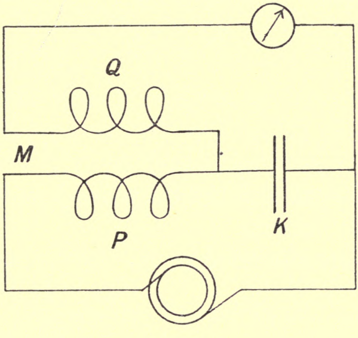


FIG. 24.

THEORY AND TEST OF VIBRATION GALVANOMETER

Using the general equation (see chapter on Damping), we have

$$\mathrm{I}\frac{d^2\theta}{dt^2}+k_1\frac{d\theta}{dt}+k_2\theta=\mathrm{GC}\sin pt,$$

where k_1 is the retarding torque at unit angular velocity,
k_2 is the control torque,
G is the galvanometer constant,
and C sin pt the current through the instrument.

Writing $\quad a=\frac{k_1}{\mathrm{I}},\quad b=\frac{k_2}{\mathrm{I}},\quad d=\frac{\mathrm{GC}}{\mathrm{I}},$

the solution is

$$\theta=\frac{d\sin(pt-\phi)}{\sqrt{(b-p^2)^2+(ap)^2}},$$

which is analogous to that for a circuit containing capacity, self-induction, and resistance.

For resonance we must have

$$b - p^2 = 0$$

or
$$\frac{1}{T} = n,$$

or the periodic time of the instrument movement equal to that of the current.

At resonance, therefore,

$$\text{Amplitude of swing} = \frac{d}{ap},$$

that is,
$$= \frac{GC}{k_1 p},$$

a result independent of I, the moment of inertia. It depends, however, on the control, which may be varied :

(*a*) By varying the tension. In this case the amplitude varies as above.

(*b*) By varying the length. In this case, so long as the bridge pieces are beyond the magnet, G remains constant and the law remains as previously, viz. deflection $\propto \frac{1}{p}$. When the bridge pieces are within the gap, G is reduced. Actually the moment of inertia is made up of two terms, viz.

$$I = I_0 + I_1 l,$$

where I_0 = moment of inertia of mirror,

I_1 = inertia of unit length of strip,

and it can be shown that the law will become either

$$d \propto \frac{A}{p^3} \text{ or } \frac{B}{p^2},$$

according as the term I_0 or $I_1 l$ is the more important in the expression for I and where A and B are constants.

Tuning Test.—It was found that a small increase in tension changed the amplitude to a large extent when near resonance.

The apparatus was arranged as shown:

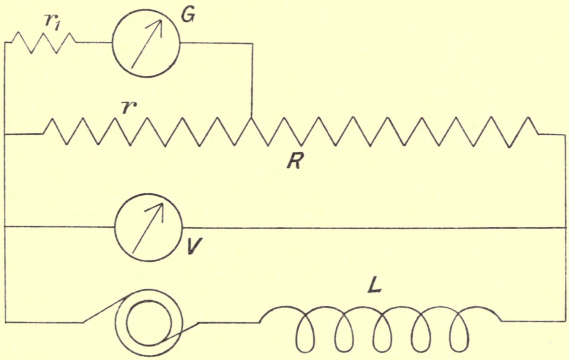


FIG. 25.

G, Vibration galvanometer.
R, r, Potential divider.
r_1, High resistance in series with G.
V, Voltmeter.
L, Choking coil to damp down higher harmonics.

The current used was 5×10^{-6} ampere very approximately, periodicity 140, distance between scale and mirror 80 cms. The upward movement of spring is given in millimetres, and the maximum deflection 23 cms. at 80 cms. is about 4·6 cms. per microampere.

Readings.—

TABLE OF RESULTS

$\frac{1}{4}$ Turns.	Millimetres moved.	Reading Cms.
0	0	2·3
1	·125	2·8
2	·25	5·0
3	·375	11·3
4	·500	22·5
5	·625	8·0
6	·75	3·7
7	·875	2·8
8	1·000	1·8
3·5	...	18·5
4·5	...	15·8

The equation

$$\text{Deflection} = \frac{23}{\sqrt{(10 - 2{\cdot}5x)^2 + 1}}$$

gave a close agreement.

×	Calculated Deflection.	Observed Deflection.	Remarks.
0	2·28	2·3	An error of a small fraction of a turn makes a great difference in the higher readings owing to steep curve.
1	3·05	2·8	
2	5·1	5	
3	9·65	11·3	
3·5	14·4	18·5	
4	23	22·5	

We see from the readings that to halve the deflection at 140 cycles only $\frac{1}{8}$ millimetre in 3 m/mms. is required, since the scale reading at maximum deflection corre-

sponds to an extension of 3·0 millimetres for direct current. Hence a 4 per cent change in a total extension of 3·0 millimetres halves the deflection. And since

$$n = \frac{1}{2\pi}\sqrt{\frac{k_2}{I}},$$

a 2 per cent change of periodicity will alter tension by 4 per cent, and amplitude by 50 per cent.

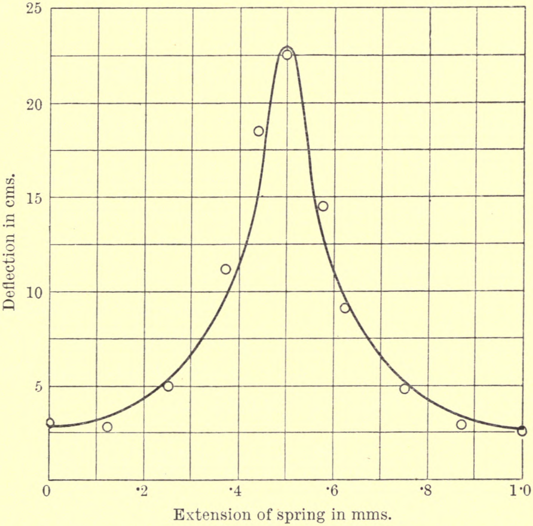


FIG. 26.—Resonance Curve. Vibration galvanometer on AC ; curve plotted from calculated values ; points observed values.

VIBRATION GALVANOMETER : TESTS WITH DIRECT CURRENT

The instrument used was of the type illustrated in Fig. 27.

The bridge pieces to vary the adjustment are movable by right- and left-hand screws.

The different adjustments give the following:

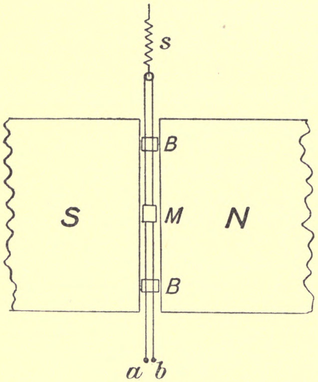


FIG. 27.

N, S, Permanent magnet.
a, b, Fixed ends of strip.
M, Mirror.
B, B, Bridge pieces.
s, Spring balance.

(1) Deflection with constant current $\propto \dfrac{1}{\text{tension}}$.

(2) (a) With bridge pieces beyond the magnet,

Deflection $\propto$ length.

(b) Bridge piece in gap,

Deflection $\propto l^2$.

The tension screw had $\frac{1}{2}$ millimetre pitch, screw for bridge pieces 1 millimetre pitch.

Since the tension is proportional to the extension of spring we have for zero reading, x then being the extension,

$$(x+0)13{\cdot}3=(x+6)4$$

or $$x=2{\cdot}56\ ;$$

thus for a scale reading s, we have

$$2{\cdot}56\times 13{\cdot}3=(2{\cdot}56+s)d,$$

from which d can be calculated.

(2) (a) With length 15·4, deflection was 16·4.
With length 10·4, deflection was 10·8.

(2) (*b*) With length 10, the deflection was 10·6, and with the scale reading 10, length 5·4, deflection was 3·1. The following curves illustrate those results :

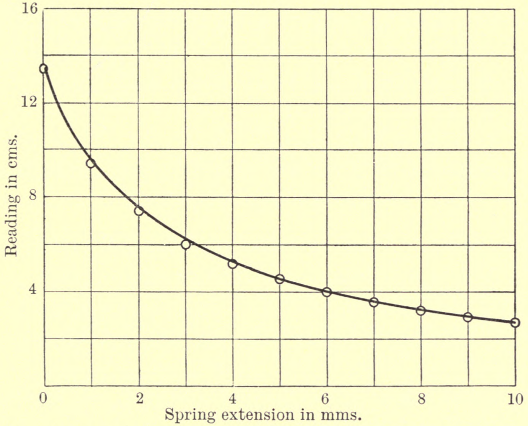


FIG. 28.—Curve illustrating effect of varying tension. Curve plotted from calculated values ; points observed values.

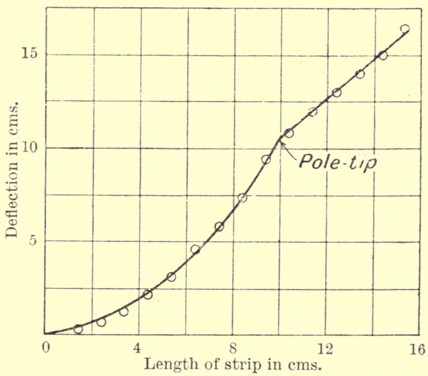


FIG. 29.—Curve illustrating relation between variation in length and observed deflection compared with calculated results on assumptions that (*a*) $d \propto l$ (bridge pieces beyond magnet) ; (*b*) $d \propto l^2$ (bridge pieces in gap). Curve plotted from calculated values ; points observed values.

Moving Coil Instruments

The following tables give some particulars of American instruments of this class taken from a report (Bureau of Standards, March 1911) by Fitch and Huber:

A Comparison of American Direct-Current Switchboard Voltmeters and Ammeters.

March 1911. Fitch and Huber. Bureau of Standards.

Voltmeters and Ammeters—

International Electric Meter Co., Chicago.
Weston, Newark, New Jersey.
Western Electric Manufacturing Co., St. Louis.
G.E.C. Schenectady.
Ridler Smith Co.
Keystone Electrical Inst. Co., Philadelphia.
Westinghouse.

The names of the makers are not given in the text, but an illustration of the instruments.

In some cases, the names on the instruments are very indistinct, no indication as to which is which being given.

Voltmeters and ammeters are in all cases by same makers when bearing same letter.

Calibration.—

Voltmeters maximum instrumental error, 1 per cent.

Ammeters (*a*) 2 per cent.
(*b*) very bad.

And the rest, maximum about 1 per cent.

Ammeters.	a.	b.	c.	d.	e.	f.	g.	h.
	%	%	%	%	%	%	%	%
Change due to Thermal E.M.F.	0	1·0	0	0	·6	·6	·1	·1
Change due to Heating .	·8	·3	·6	·6	1·1	0	·2	·3
Sum .	·8	1·3	·6	·2	1·7	·6	·3	·4
Observed .	·5	1·2	·9	·1	1·6	·4	·3	·6

Damping.—Aluminium formers used.

Stray Field.—The movement was placed at the centre of a coil having two turns—62·5 cms. diameter, carrying 100 amperes ; the flux across gap was 2 gausses. This is about 5 times earth's total field—10 times when reversed—20 times H, or the same as 500 amperes in a straight bar at 25 cms.

Small fields perpendicular to gap produce a small effect.

Temperature Coefficients.—

Spring, $-0·04$ per cent per degree C.

Magnet may be + or −, usually same order and sign as spring.

Wire should be practically nil in voltmeter.

[TABLE

Voltmeters—mostly 150 Volts

	a.	b.	c.	d.	e.	f.	g.	h.
Resistance	·12450	·16180	·18400	·7540	·12350	·14380	·16760	·14190
Watts at 150 volts . .	1·81	1·39	1·22	2·99	1·82	1·56	1·34	1·58
% change 1 hour, 150 volts in reading . . .	– ·2 [1]	– ·2	– ·1	– ·1	– ·3	0	+ ·1	– ·2
% change reversal of 4 gausses	·8	2·2	1·5	1·1	2·5	2·3	1·2	1·7
Damping secs. to come to rest, 120 volts . .	·8	4·8	2·1	1·2	4·6	2·4	2·8	3·7
Balance % of max. reading	0	·4	·2	·1	·3	·1	·1	·1
Megohms insulation resistance	>75	15	37	>75	75	>75	25	>75
Temperature coefficient % per deg. C. . . .	– ·01 [1]	– ·03	– ·03	– ·02	– ·02	– ·01	+ ·02	– ·01

[1] The negative sign means fewer volts required.

Ammeters—200 Amperes

	a.	b.	c.	d.	e.	f.	g.	h.
Resistance of movement	1·3	3·7	2·7	·9	2·0	1·3	4·8	3·2
Shunts: Full load—								
Drop-millivolts	50	74	60	49	70	50	61	97
Watts	10	14·8	12	9·8	14	10	12·2	19·4
Temperature in plates	53	73	69	38	70	66	62	82
Temperature in lugs	49	59	58	20	45	58	50	43
Thermal E.M.F. millivolts in one hour	0	·7	0	0	·4	·3	·1	·1
% change of resistance 25°-50° C.	·5	·1	·2	·1	·4	0	·1	·1
% change one hour in the shunt	– ·5	– 1·2	– ·9	– ·1	– 1·6	– ·4	– ·3	– ·6
% change – 4 gausses	·7	1·5	1·4	1·1	3·5	1·6	·8	1·9
Damping – 160 amps.	1·1	2·6	1·6	1·4	5·8	2·1	2·4	2·4
Balance	·9	·7	·7	·4	·6	·6	·8	·7
Megohms insulation resistance	75	75	37	75	>75	>75	9	75
+ temp. coefficient	·11	·08	.09	·15	·15	·32	·28	·20

MOVEMENTS

	Grammes.	Torque, grm. × m/m.	$\frac{I}{W}$.	C Milliamperes.	Turns.	A.T.
A	1·72	9·4	5·5	12	69·5	·83
B	3·02	2·9	1·0	9·3	53·5	·50
C	1·64	5·0	3·0	8·1	58·5	·47
D	2·82	21·0	7·4	19·9	103	2·06
E	2·88	8·8	3·0	12·1	155·5	1·88
F	3·48	15·0	4.3	10·4	195·5	2·04
G	1·73	2·9	1·7	8·9	45·5	·41
H	3·50	8·6	2·5	10·6	116·5	1·24
a	2·55	6·2	2·4	37	11·0	·40
b	4·66	3·7	·8	20	24·5	·49
c	2·17	3·7	1·7	23	12·4	·29
d	3·26	7·6	2·3	53	13·3	·71
e	3·71	3·1	·8	36	22·5	·81
f	4·36	6·7	1·5	38	26·5	1·01
g	2·70	2·1	·8	13	21·5	·28
h	4·70	8·7	1·9	13	36·5	1·13

Remark.—Turns may be in parallel.

MAGNETS

	Length of Magnet in cms.	Section of Magnet in sqr. cms.	Polar Area in sqr. cms.	Gap Length in cms.	K.	H Gap.	Kilo. Lines.	B in Magnet.
a	26·4	4·38	11·0	·260	255	1590	17·5	4000
b	25·9	3·94	10·9	·293	246	427	4·66	1180
c	34·7	3·02	11·2	·255	505	1450	16·2	5400
d	22·4	3·90	9·1	·173	302	2790	25·4	6500
e	30·4	2·93	10·6	·450	244	213	2·25	770
f	23·7	2·50	11·0	·287	363	875	9·63	3800
g	27·7	4·01	8·8	·310	196	551	4·85	1210
h	24·7	4·70	13·7	·320	225	740	10·1	2200

$$K = \frac{\dfrac{\text{length of magnet}}{\text{section of magnet}}}{\dfrac{\text{length of air gap}}{\text{section of air gap.}}}$$

Heinrich and Bercovitz in article on "Die technischen Messinstrumenter," in the *Handbuch der Elektrotechnik*, call $\frac{K}{100}$ "factor of safety" against demagnetisation.

Shunts and Compensating Devices

These when used with permanent magnet moving coil instruments are so arranged that when carrying their maximum current the volt drop is sufficient to send the pointer of the instrument to the end of its range.

If it is made of a strip of material, specific resistance ρ, breadth b, length l, and thickness t, we have

$$v = C\rho l/bt \quad . \quad . \quad . \quad \text{(i.)}$$

But generally a certain number of watts per unit of surface are supposed to be dissipated,

$$\therefore \quad \omega = 2blk \quad . \quad . \quad . \quad \text{(ii.)}$$

The thickness of the sheet, say of manganin is generally known, so that from (i.) we have

$$\frac{l}{b} = \frac{v}{C} \cdot \frac{t}{\rho}$$

and from (ii.) $$bl = \frac{\omega}{2k},$$

$$\therefore \quad l^2 = \frac{v}{C} \frac{t}{\rho} \cdot \frac{\omega}{2k}$$

and
$$b = \frac{\omega}{2kl},$$

so that the approximate length and breadth can be prearranged when k is known.

If we have a number of strips in parallel, t will be their total thickness. The length "l" usually lies between certain limits influenced by considerations of cost and space taken up.

COMPENSATING DEVICE FOR AMMETER [1]

Consider the arrangement shown in Fig. 30 with

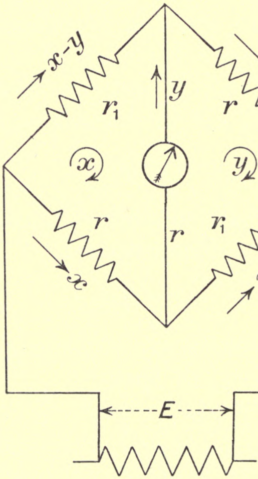


FIG. 30.

the network currents x and y, and let E be the volt drop across the shunt.

[1] See a paper by Edgcumbe and Punga, *Proceedings I.E.E.*, March 24, 1904.

Then we have

$$rx + ry - r_1(x - y) = 0 \quad . \quad . \quad . \quad \text{(i.)}$$

$$r(x - y) + rx = \mathrm{E} \quad . \quad . \quad . \quad \text{(ii.)}$$

Multiplying (i.) and (ii.) by $r + r_1$ and $r - r_1$ respectively, we obtain

$$(r^2 - r_1^2)x + (r + r_1)^2 y = 0,$$

$$(r^2 - r_1^2)x - r_1(r - r_1)y = \mathrm{E}(r - r_1).$$

$$x(r^2 - r_1^2) - r_1 y(r - r_1) = \mathrm{E}(r - r_1).$$

From these

$$y = \frac{\mathrm{E}(r_1 - r)}{r(r + 3r_1)}.$$

Let r become $r + \epsilon$ through temperature rise, then

$$y = \mathrm{E}\frac{r_1 - (r + \epsilon)}{(r + \epsilon)(3r_1 + r + \epsilon)}.$$

If $y = \mathrm{Y}$, the current through the instrument unchanged by temperature rise,

$$\frac{r_1 - r}{r(3r_1 + r)} = \frac{r - (r + \epsilon)}{(r + \epsilon)(3r_1 + r + \epsilon)}.$$

Simplifying and neglecting $\epsilon(r_1 - r)$, we finally arrive at the result that

$$3r_1 - r = 0.$$

Hence for compensation

$$r_1 = \frac{r}{3}.$$

Let the ordinary current through a shunted ammeter be

$$y = \frac{\mathrm{E}}{r},$$

and with this device

$$y_1 = \frac{E}{r} \frac{r_1 - r}{3r_1 + r},$$

or

$$\frac{y}{y_1} = 3.$$

With ordinary connections, and 20° C. temperature rise, we have

$$\frac{Y}{y} = 1 \cdot 05.$$

With compensating device we have ratio

$$\frac{Y_1}{y_1} = \frac{E r_1 - r + \epsilon}{(r+\epsilon)(3r_1 + r + \epsilon)} \Big/ \frac{E}{3r},$$

or inserting values

$$\frac{Y_1}{y_1} = \frac{3 \cdot 0035}{3 \cdot 000}.$$

We see that with the compensation device the error, owing to temperature changes upsetting ratio of shunt and coil resistance, is reduced from 5 per cent to about 0·1 per cent, which is a matter of considerable importance under certain conditions of testing.

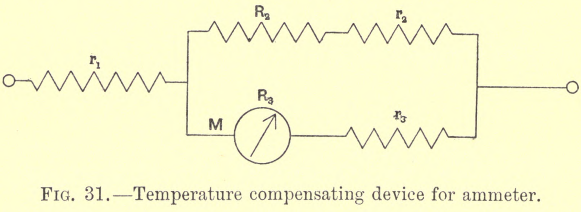


FIG. 31.—Temperature compensating device for ammeter.

In the arrangement indicated in Fig. 31 the capitals

indicate resistances of copper, the small letters resistances with negligible temperature coefficient.

The condition to be satisfied is that for a given constant volt drop e across the shunt the current through the movement R_3 shall be constant and irrespective of temperature variations.

It follows that neglecting very small quantities

$$\frac{(r_1+r_2+R_2)(r_3+R_3)}{r_2+R_2}=(r_3+R_3)R_2+R_3(r_2+R_2+r_1),$$

which is a simple equation in r_1. Hence, by arbitrarily choosing the other resistances, r_1 can be found.

Voltmeter and Ammeter Windings

In permanent magnet instruments of the moving coil variety several questions arise, viz. what are the limits regarding ampere turns on ammeters and voltmeters, what is the best size of wire and shape of coil, and finally, what effect has a change of the dimensions on the working ?

Best Size of Wire.—

Let v = volts across coil.
l = mean length of one turn.
b = the breadth of former.
n = number of turns.
i = current through windings.
R = resistance of windings.
R_1 = resistance of leads and springs.
a = radius of wire.
I = moment of inertia of coil.
G = galvanometer constant.

We see that

$$\text{Resistance} = \frac{\rho l n}{\pi a^2}, \quad \text{Current} = \frac{v \pi a^2}{\rho l n},$$

$$\text{Ampere turns} = \frac{v \pi a^2}{\rho l}.$$

In an ammeter we require maximum ampere turns for a given voltage drop. This is obtained by making a as large as possible.

The limits are: (a) The air gap must be kept small; (b) springs have an appreciable resistance.

Hence the expression for current becomes

$$\frac{v}{\frac{\rho l n}{\pi a^2} + R_1},$$

and

$$\text{Ampere turns} = \frac{vn}{\frac{\rho l n}{\pi a^2} + R_1}.$$

In order that this quantity be a maximum for a given value of v its reciprocal must be a minimum.

Let

$$y = \frac{\rho l n}{\pi a^2 v n} + \frac{R_1}{vn},$$

$$= \frac{\rho l}{\pi a^2 v} + \frac{R_1}{vn}.$$

In this a and n are variables, n being a function of a.

Approximately $n = b/\text{diameter of insulated wire}$, or

$$n = \frac{kb}{2a}.$$

The quantity k corresponds to the usual space factor.

Hence
$$y=\frac{\rho l}{\pi a^2 v}+\frac{R_1 2a}{vbk}.$$

Differentiating and equating to zero, we find finally that
$$R=\frac{R_1}{2},$$

or if the resistance of the coil be one-half that of the external resistance, the ampere turns are a maximum for a given volt drop. If a series resistance of low temperature coefficient be employed, R_1 = resistance of springs + leads + the external resistance.

Now with regard to the voltmeter.

Here the v becomes E the voltage, and the usual expression for size of wire is
$$a=\sqrt{\frac{\text{Ampere turns}\times\rho l}{\pi E}}.$$

This expression gives much too fine a wire for practical purposes and consequently to reduce this and diminish temperature errors a series coil is used. In one case, the resistance of the coil was about 75 ohms, total resistance 13500 ohms, range 0–150 volts.

Best Shape of Coil.—Assuming flux density, number of turns and area of coil constant. Hence area $=\rho l$ for a rectangular section of the coil. The minimum resistance is given by minimum perimeter, *i.e.* a square.

Effect on Damping.—The ratio of successive swings has been shown (see chapter on Damping) to be G^2/RI where R is the resistance of the former, or $R=2(b+l)k_1$, and if m is the mass per unit length,
$$RI=2k_1 m\left\{\frac{b^2}{4}+b^2A+\frac{b^2A}{24}+A^2\right\}.$$

We may differentiate this with respect to b, and we see it can only equal zero when b is diminished indefinitely.

Hence we see that RI diminishes and the ratio of successive swings increases.

Limits are : (a) I of pointer or mirror gives a minimum value ; (b) if

$$\frac{k}{\mathrm{I}}-\frac{1}{4}\left(\frac{\mathrm{G}^2}{\mathrm{RI}}\right)^2$$

be positive the motion is dead beat. Hence we see for maximum sensitiveness a square section is best, for best damping a narrow rectangle.

Alteration of Dimensions.—Let the length and breadth be increased x fold, everything else being constant. It is easily seen that the figure of merit will alter from

$$\frac{100\,.\,\theta\,.\,k}{\mathrm{IR}^{0\cdot4}} \text{ to } \frac{100\,.\,\theta\,.\,k}{\mathrm{IR}^{0\cdot4}}\times\frac{1}{x^{2\cdot4}},$$

and will thus decrease if x be made $>$ unity.

Comparison of Square and Circular Section.—Assume constant control flux density and area of coil. Then the ratio of deflections will be

$$\frac{\text{Square}}{\text{Circular}}=\frac{\pi}{4\cos\phi}.$$

$\pm\phi$ are the limits of integration in finding torque on circular section—so there is not much difference.

Ratio of Resistances.—This will be

$$\frac{2}{\sqrt{\pi}}.$$

Sensitiveness.—Suppose you have a circular, and square section, as an ammeter. Then for deflections

$$k\theta = \frac{Hb^2n}{10} \cdot \frac{e}{4bR_1},$$

$$k\theta_1 = \frac{Hnr^2}{10} \frac{e \, . \, 4 \cos\phi}{2\pi r R_1},$$

R_1 being resistance per unit length. Then since $b^2 = \pi r^2$, we have

$$\frac{\theta}{\theta_1} = \frac{\pi^{3/2}}{8 \cos\phi}.$$

Apparently, therefore, a circular section is the better, since $\cos\phi = 0{\cdot}7$ approximately.

Galvanometer Suspensions and Sensitiveness

Suspensions.—A galvanometer needle or coil is generally suspended by means of a quartz fibre, metal strip, or some bifilar suspension.

In the latter case we might write

$$\text{Deflection} \propto \frac{L}{D \, . \, W(1 - k)}$$

where L is the length of suspension,

D the product of distance between them at the top and bottom,

W the weight supported.

k is a constant depending on the difference of tensions in the suspensions.

We can, therefore, increase the "sensitiveness" by making L large, and any one or the product of D.W.$(1 - k)$ small.

For a round wire length L, diameter d,

$$\theta \propto l/d^4,$$

and for a flat strip breadth b, thickness t,

$$\theta \propto l/bt^3,$$

if t is small compared with b. By making t small the sensitiveness can be increased, and in addition the zero is found to be more constant than with a round section of wire, besides giving considerable mechanical strength. In ammeter and voltmeter movements the springs are generally of the flat strip variety of phosphor bronze.

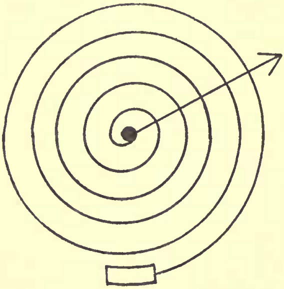


Fig. 32.

When a spring is rigidly fastened at one end (Fig. 32), and its convolutions fairly numerous (more than ten generally give satisfactory results), and the centre of the spring at the centre of gravity, it can be shown that the

$$\text{Deflection} \propto \text{length}$$

for a given torque winding or unwinding it. If the spring is not truly circular or centred, then the law, though still a linear one, has a constant term; the

$$\text{Deflection} \propto \text{length} - \text{constant}$$

in this case.

Effect of Iron Case of Instruments.—Iron cases support the movement of switchboard type instruments, and although acting as magnetic shields have some disadvantages.

In moving coil instruments the cover acts as a magnetic shunt to the magnet, and in one case the instrument read about 5 per cent lower with the cover in position.

In moving iron instruments, on the other hand, the residual magnetism of the case causes the deflections to differ with reversed polarity, otherwise there is no great objection to it.

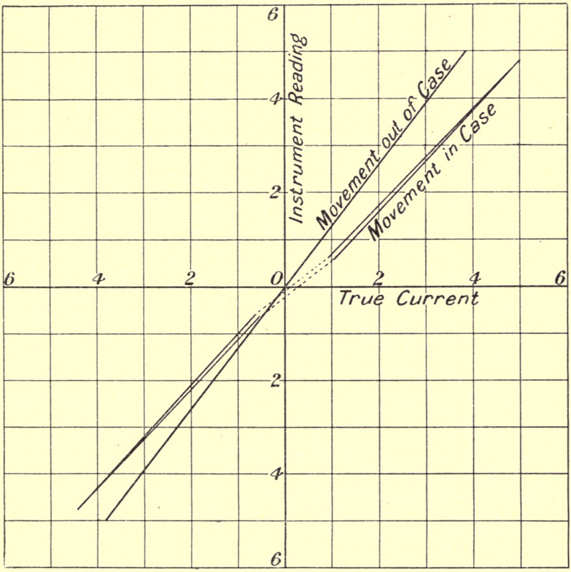


Fig. 33.—Instrument of moving iron type compared with moving coil instrument curve: (*a*) in case; (*b*) without case. In latter case an experimental coil was used, which accounts for the higher readings.

The curves (Figs. 33, 34) illustrate this on direct and alternating current and with reversed polarity.

Figure of Merit.—This is generally defined as

$$\frac{100 \times \text{deflection per microampere in m/mms. at a metre}}{t \times R^{0 \cdot 4}},$$

t being the undamped periodic time, R being the resistance.

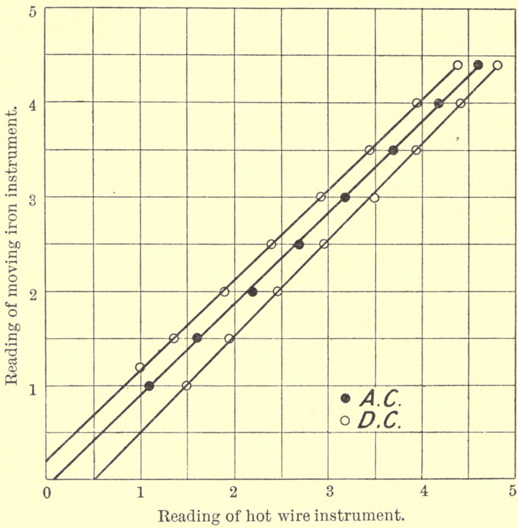


FIG. 34.—The same instrument was compared on both D.C. and A.C. with a hot wire instrument.

Sensitiveness is usually referred to either as $\frac{\theta}{i}$ or $\frac{d\theta}{di}$. Instruments of the older reflecting pattern possessed the advantage of varying sensitiveness by neutralising the earth's control by varying the positions of the controlling magnet. D'Arsonval galvanometers are not generally arranged with magnetic shunts.

In galvanometers of the tangent type

$$d\theta = \frac{di}{i} \cdot \frac{\sin 2\theta}{2} \cdot$$

If $d\theta$ is to be a maximum for a given change di in current, then

$$\theta = 45°.$$

For a moving coil instrument

$$d\theta = \frac{di}{i} \cdot \theta\,;$$

therefore to detect a change di with maximum accuracy, θ should be as great as possible.

Design of Moving Coil Instruments.—For the design of these for use with the deflection potentiometer, or similar purpose, the reader should consult *American Bureau of Standards*, H. B. Brook's "Outline of Design," vol. viii. No. 2, p. 419, from which the following is taken :

Pivoted instruments are preferable to reflecting moving coil types, since (*a*) less difficult to read, (*b*) better scale owing to stronger spring control, (*c*) no necessity for accurate levelling, (*d*) vibration has less effect.

The value of the gap flux can be measured either by noting the throw on ballistic by means of a search coil or passing a known current through the coil of the instrument (as in the Koepsel permeameter).

The torque due to control was measured by hanging weights at a given radius, and keeping the pointer at its maximum value horizontal.

Instruments for use with deflection potentiometers

should satisfy certain conditions; t should be 1 to 1·5 seconds. If R is coil resistance, R^1 the critical resistance,

$$\frac{R^1}{R}=5 \text{ to } 10.$$

iR^1 must have a given value. The value of k must also satisfy the condition that

$$\frac{\text{Torque in grms. cms.}}{\text{Weight in grms.}}$$

be not less than $\frac{5}{100}$ for a 90° movement.

If R^1 is the critical resistance, then

$$H=A\sqrt{\frac{R^1}{Rt}},$$

where A is a nominal constant (see W. P. White, "Everyday Problems of the Moving Coil Galvanometer," *Phy. Review*, No. 23, p. 384, 1906). The constant A follows from the condition for critical resistance, viz.

$$\left(\frac{G}{RI}\right)^2-4\frac{k}{I}=0.$$

See chapter on Damping.

Since $Ri \propto H$, if H_1 is changed to H_2, we have

$$H_2=H_1\sqrt{\frac{R_1R_2{}^1}{R_1{}^1R_2}}.$$

If a coil of N turns be wound to have xN turns, the coil resistance $=x^2R$, and critical resistance x^2R^1; current

per division $=\frac{i}{x}$, and product current per division $\times$ critical resistance $= x\mathrm{R}^1 i$.

If we change turns from N_1 to N_2,

$$\mathrm{N}_2=\frac{i_2\mathrm{R}_2}{i_1\mathrm{R}_1}\sqrt{\frac{\mathrm{R}_2\mathrm{R}_1{}^1}{\mathrm{R}_2{}^1\mathrm{R}_1}}\times\mathrm{N}_1,$$

and if D_1, D_2 are the diameters of the wire,

$$\mathrm{D}_1/\mathrm{D}_2=\sqrt{\frac{\mathrm{N}_2}{\mathrm{N}_1}}.$$

So we finally have

$$\left(\frac{\mathrm{D}_2}{\mathrm{D}_1}\right)^2=\frac{i_1\mathrm{R}_1}{i_2\mathrm{R}_2}\sqrt{\frac{\mathrm{R}_2{}^1\mathrm{R}_1}{\mathrm{R}_2\mathrm{R}_1{}^1}}.$$

If for $i_2\mathrm{R}_2$ the desired volt drop is substituted, and $i_1\mathrm{R}_1$ the calculated volt drop per division in the actual instrument, D_1, the observed value of size of wire, will give D_2, the size to use.

Then calculate N_2 turns on coil, and find $\frac{\mathrm{R}_2{}^1}{\mathrm{R}_2}$; solve for H_2 from equation above, and adjust by shunting.

In this a certain amount of trial and error comes in, and the size of wire is only approximate, the nearest being used.

To cheapen the cost of manufacturing galvanometers having a specified critical resistance, the following is recommended:

(1) Tested springs of slightly under normal strength be used.

(2) Magnet strength adjusted by shunting till critical resistance about correct.

Using this critical resistance, make a scale to fit the instrument by applying varying voltages.

Other methods of adjusting are by altering the moment of inertia and damping, etc. ; for these the paper referred to must be consulted.

CHAPTER IV

IRON CORED INSTRUMENTS

INSTRUMENTS with iron cores have been used from time to time, and recently Dr. Sumpner has proposed to use iron cored ammeters, voltmeters and wattmeters for measuring alternating currents, volts and watts.

The original papers will be found in the *Journal of the I.E.E.* vols. xxxiv. and xxxvi., and a paper on " New Alternate Current Instruments " (in vol. xli. p. 227) by Messrs. Sumpner and Record.

Figs. 35, 36, 37, 38 show the general arrangement of the instruments.

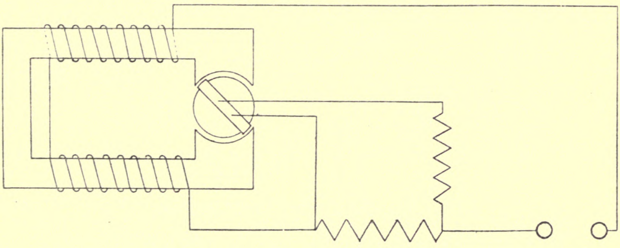


FIG. 35.—Ammeter series magnet.

In designing such instruments, the chief difficulties appear to be as follows :

1. Eddy currents may influence the magnitude of the field of force on the gap.

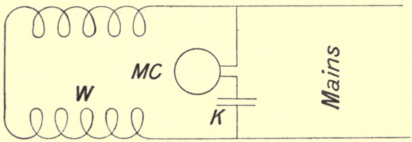


FIG. 36.—Voltmeter shunt magnet.

MC, Moving coil. W, Magnet winding. K, Condenser.

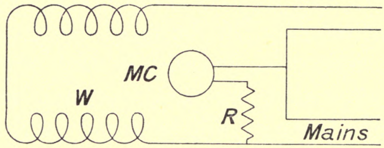


FIG. 37.—Wattmeter series magnet.

MC, Moving coil. W, Magnet winding. R, Resistance.

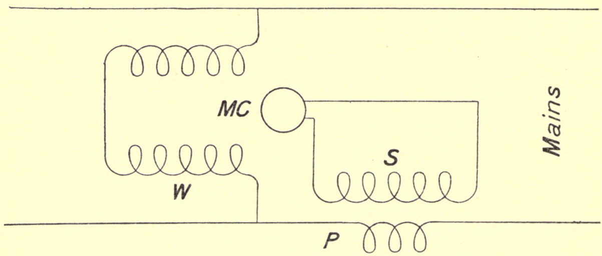


FIG. 38.—Wattmeter shunt magnet.

P, S, Quadrature transformer. MC, Moving coil.
W, Magnet winding.

2. Hysteresis introduces an angle of lead between current in the moving coil and gap flux; or varying

frequency and wave form may introduce errors, owing to the reactance of the moving coil circuit.

The former of these merely means an alteration of the calibration constant for ammeters and voltmeters, which is not important, but for a wattmeter some auxiliary apparatus, such as an air gap transformer, is used to overcome leakage flux distribution under the conditions necessary to eliminate hysteresis error.

With the voltmeter and ammeter arranged with series magnet, then if Z is the flux and i the moving coil current, the torque

$$T \propto Zi,$$

and if there is a phase angle θ, then

$$T \propto Zi \cos \theta.$$

Z is proportional, as also is i, to the main current, or voltage I_0, therefore

$$T \propto I_0^2 \cos \theta.$$

If there is a hysteresis angle or frequency effect such that

$$I_1^2 \cos \theta_0 = I_0^2 \cos (\theta - \theta_0),$$

or if

$$\theta_0 = 0,$$

$$I_1 = I_0\left(1 - \frac{\theta^2}{4}\right),$$

$$\therefore \quad \frac{I_0 - I_1}{I_0} = \frac{\theta^2}{4}.$$

The instrument will, therefore, read correctly to 1 per cent if θ is less than $\frac{1}{5}$th of a radian.

In a wattmeter, however, the ratio of actual to true reading is

$$\frac{\cos(\phi - \theta)}{\cos \phi},$$

and for small values of θ this gives an error

$$\theta \tan \phi.$$

Now this error is serious since it varies with θ.

When a series magnet is used with a wattmeter it is difficult to bring down the hysteresis phase error to 1 per cent, but with a shunt magnet, and the smaller the gap the less this error will be, and a current transformer may be used with the moving coil, the total phase error will be

$$\theta = \theta_r + \theta_s + \theta_t,$$

where

θ_r = is angle due to resistance,
θ_s = is angle due to inductance of moving coil,
θ_t = is angle due to transformer.

It will be noticed that if a wattmeter of the shunt

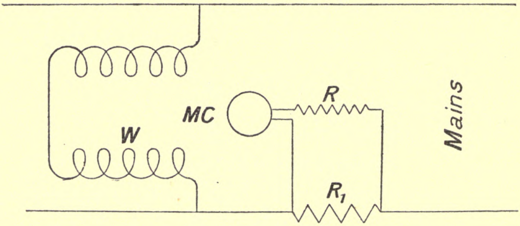


FIG. 39.—Idle current ammeter.

MC, Moving coil. W, Magnet winding.
R_1, Shunt. R, Series resistance.

magnet type be arranged as in Fig. 39 it will give no deflection on a non-inductive load, but will read

$$VI \sin \phi,$$

and for circuits of constant voltage and frequency will read the "idle current" (*vide* also "The Theory of Phase Meters," Sumpner, *Proceedings Phys. Soc.*, Oct. 27, 1905).

The important question as to whether series or shunt magnets are the more suitable seems to be settled in favour of the shunt magnet when possible. With a series magnet the magnetism excited is dependent on the properties of the iron, whereas with a shunt magnet the *total* magnetism is dependent only on the voltage, and only the magnetising current is affected by permeability changes. The field of the shunt magnet is not in phase with the voltage, and a special form of current transformer is needed; or for a voltmeter connection, a condenser is used.

The strong field excited giving a torque of 1·2 grm. cms. enables strong controlling springs to be used.

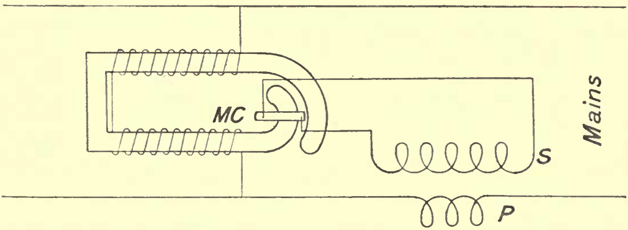


Fig. 40.

MC, Moving coil. P, S, Quadrature transformer.

The wattmeter then consists of a shunt magnet system made up of stampings as shown with moving

coil connected to a quadrature transformer. This quadrature transformer consists of a primary and

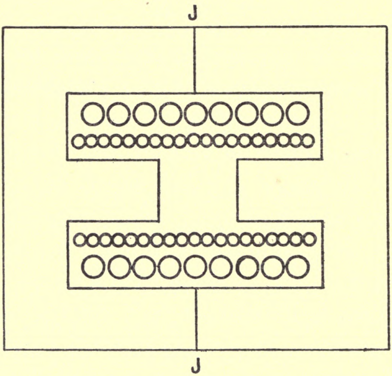


FIG. 41.—Quadrature transformer.

J, J, Butt joints.

secondary winding arranged as shown in a series of stampings with an air gap. This transformer produces a secondary current directly proportional to the primary current for all values of the latter, but this current is also 90° out of phase with the primary current. The core is formed of two sets of stampings with butt joints at JJ. Owing to the large gap and fact that the permeability of the iron is about 1000 for inductions used, the magnetic reluctance is chiefly that due to the gap, and about 100 times that of the core. Therefore it is independent of all the magnetic variations in the core to that extent.

For large currents other types of transformers are made, and potential transformers used for high tension work.

Since the current passing in the moving coil circuit depends on $\frac{dI}{dt}$ where I is the primary current and the flux depends on the instantaneous voltage, we see that the instrument measures

$$\frac{1}{T}\int_0^T\left(\int_0^T V dt\right)\frac{dI}{dt}dt = k\theta,$$

where k is a constant, θ the angle of deflection, and T is the periodic time. The readings are therefore independent of wave form or frequency. This is at once obvious, since at any instant

$$\frac{dI}{dt} = pI$$

where $$p = 2\pi n$$

and $$\int V dt = \frac{V}{p}.$$

This relation is true over a great range of frequency.

Power Factor.	% of Volt Amperes. 100 θ sin ϕ.
1·0	0
·9	0·43
·7	0·71
·5	0·87

When $\theta = 0{\cdot}01$ the error table above was calculated. From which it will be seen that the error is less than 1 per cent even at such low power factor as 0·5.

A paper on "Alternate Current Measurement," by Dr. Sumpner (*Proceedings of Royal Society*, vol. lxxx. p. 310), should also be consulted regarding the error due to the induced current in the moving coil. This induced current is due to the action of the shunt magnet upon it. While this current is out of phase by 90° with the flux producing it, it has considerable magnitude. The fact that it is out of phase by 90° leads to the result that no torque is due to it, if the self-induction of the moving coil is negligible. In other cases naturally a torque must result from the component in phase with the driving current in the moving coil.

Moving Iron Instruments

Although it is preferable to use instruments of the permanent magnet moving coil type, still when great accuracy is not required, combined with cheapness, these instruments are much used. (This applies to D.C. instruments.)

Probably the best type of soft iron instrument was Lord Kelvin's Ampere Gauge. Here a solenoid sucked down a plunger of soft iron and deflected a pointer over a scale. The soft iron was kept "saturated" magnetically by means of a fine wire coil, so that the movement was regulated by the current only. If residual magnetism was present, the readings on first increasing and then diminishing the current generally differ to some extent.

The action of a coil on its plunger has been investigated by Breguet (see *Cantor Lectures* on "The Electro-Magnet," by S. P. Thompson, F.R.S.).

In other types of these instruments, the needle of soft

iron is caused to move from a weaker to a stronger field by being arranged so as to move eccentrically relative to the coil, or a fixed piece of magnetised iron repels a moving piece of iron. In such a case the control is a spring or gravity, and as a rule the graduations are not uniform over the range.

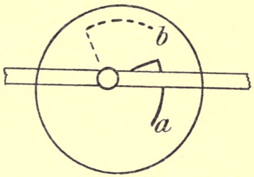


FIG. 42.—Moving iron instrument.
a, Undeflected. *b*, Deflected.

In instruments of this class for measuring volts or amperes the action depends on

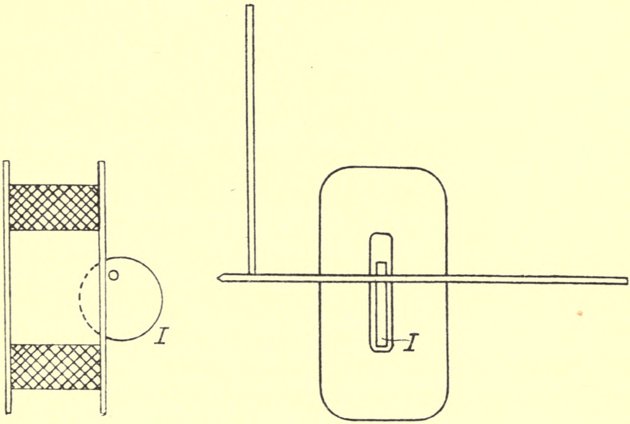


FIG. 43.—Moving iron instrument.
I, Soft iron discs.

the field of force due to the coil. For any solenoid the force at its centre is proportional to

$$nI,$$

the ampere turns.

Suppose, for instance, an ammeter read over its whole range for a maximum current of 200 amperes, and that there were three turns on the coil. The ampere turns then are 600. If we wish to re-wind the instrument and use it on a 100 volt circuit as a voltmeter, we must have

$$c \times n = 600$$

if the range is to be the same. But

$$c = \frac{V}{r} = \frac{V\pi d^2 n}{4sl} = 600.$$

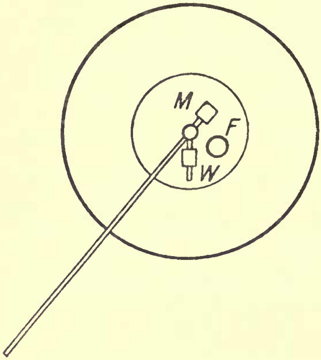


FIG. 44.—Moving iron instrument.
M, Moving iron. F, Fixed iron.
W, Balance weight.

But if the mean length of a turn is M, then $l = n$M,

$$\therefore \quad d = \sqrt{\frac{600 \times \text{M} \times 4 \times s}{\pi \text{V}}},$$

assuming no ballast resistance be used, so that knowing the mean length of a turn, the specific resistance of the material and the voltage, the diameter of the wire is determined. The voltmeter must be of relatively high resistance, and in all these instruments the whole current to be measured passes through the coil. In these instruments we assume that $\text{V} = cr$, and in reality it is c that is measured on the voltmeter, and the proportionality is assumed.

As a rule these instruments, owing to the weight of the needle and pointer, are not dead beat, and have

generally a dash pot to steady and damp the oscillations of the pointer.

The ammeters are rarely shunted. When an instrument of this sort is shunted, we get for the total current

$$\frac{e(s+g)}{sg},$$

s and g being the resistance of the shunt and instrument respectively.

But the instrument current is $\frac{e}{g}$,

hence
$$\frac{i}{\mathrm{I}}=\frac{g+s}{s},$$

or
$$i=\mathrm{I}\left(\frac{g+s}{s}\right)=\mathrm{I}\left(1+\frac{g}{s}\right),$$

i being the instrument current, I the total current.

We see that in an ammeter g is always small and s has to be smaller, so that we have to adjust the two small quantities $\frac{g}{s}$ accurately. This is a matter of some difficulty. We see, however, that temperature errors cancel out if the shunt and coil are made of the same materials, and temperature rise is same, which is not necessarily the case with external shunt. On low voltage circuits, shunting might alter the total current, and necessitates insertion of resistance to equalise the difference between

$$g \text{ and } \frac{sg}{s+g}\cdot$$

If the shunt is of different material, having a temperature coefficient a for resistance, then

$$i = \mathrm{I}\left\{1 + \frac{g}{s}(1 - at^0)\right\}.$$

Effect of Shunting.—Let l mean length of one turn, n = number of turns, A the cross section of the wire. Then the question arises as to whether there is any advantage in shunting instruments of this type. For instance, does shunting increase volt drop, volume of copper or power wasted ?

Calculation shows that the watts wasted in the instruments

$$= i^2 r$$

$$= \frac{i^2 \rho l n}{\mathrm{A}} = \frac{i}{\mathrm{A}} \cdot i n l \rho.$$

In instruments of different ranges, but of the same type, the ampere turns must be constant, and if the winding space is the same, the radiating surface is constant. Therefore, the maximum watts dissipated is constant, and it follows

$$\frac{i}{\mathrm{A}} = \text{constant}.$$

The volt drop in the instrument is ir, or

$$\frac{i \rho l n}{\mathrm{A}} = \frac{i}{\mathrm{A}} \cdot \frac{ni}{i} \rho l.$$

The terms $\frac{i}{\mathrm{A}}$, ni, ρ, l, are all constant,

$$\therefore \quad \text{volt drop} \propto \frac{1}{i}.$$

The volume of copper

$$=\mathrm{A}ln,$$

$$=\frac{\mathrm{A}}{i}\,.\,nil=\text{constant}.$$

Suppose we compare two instruments: (*a*) unshunted, (*b*) shunted, the range current density, ampere turns and mean length of a turn being the same in both cases.

For (*a*)

$$\text{Volt drop}=nc\times\frac{\mathrm{C}}{\mathrm{A}}\times\frac{\rho l}{\mathrm{C}}\cdot$$

$$\text{Volume of copper}=\frac{\mathrm{A}}{\mathrm{C}}\,.\,\mathrm{C}n\,.\,l.$$

$$\text{Power wasted}=\frac{\mathrm{C}}{\mathrm{A}}\,.\,\mathrm{C}n\,.\,\rho l.$$

For (*b*)

$$\text{Volt drop}=ni\,.\,\frac{i}{\mathrm{A}}\cdot\rho l\,.\,\frac{r+\mathrm{R}}{\mathrm{R}}\mathrm{C},$$

which is greater than in (*a*) in ratio $\dfrac{r+\mathrm{R}}{r}\cdot$

$$\text{Volume of copper}=\frac{\mathrm{A}}{i}\,.\,inl,$$

which is the same as in (*a*).

$$\text{Total power wasted is}=\frac{i}{\mathrm{A}}\,.\,inl\rho+(\mathrm{C}-i)^2\mathrm{R}\,;$$

the first term is the same as in (*a*).

It would appear, therefore, on shunting an ammeter, or the series coil of a watt hour meter, that:

(*a*) The volt drop is increased.

(*b*) The volume of copper required in the instrument is the same.

(*c*) The power wasted in the instrument is the same.

We see also that there is an increase in power wasted if a shunt is used, and the extra cost of the shunt has to be taken into account.

It seems that there is little advantage in shunting soft iron instruments except, perhaps, in cases where the maximum ampere turns are so large as only to involve a fraction of a turn of winding, which might be difficult to adjust.

In moving coil instruments the advantages of shunting are obvious.

Generally speaking, the usual well-known winding relations applied to galvanometers apply to instrument coils.

For instance, the rate of heat production in a coil is a constant for a given magnetic effect and independent of gauge of wire used.

It follows also that since resistance $\propto \dfrac{\text{length}}{\text{area}}$, then for a given winding space we double the length; we also halve the area of the wire,

$$\therefore \quad R \propto n^2.$$

But magnetic effect depends on nC or ampere turns. Hence magnetic effect depends on

$$\sqrt{R} \,.\, C$$

for a given current. If θ be some given deflection, then

$$\sqrt{R} \,.\, C = k\theta,$$

$$C = \frac{k\theta}{\sqrt{R}} \text{ for an ammeter.}$$

For a voltmeter $$C = \frac{V}{R},$$

or $$V = \sqrt{R} \times k\theta.$$

Coils also which act on a magnetic needle should be wound so that the axial magnetic force $f = \frac{a}{r^3}$ for all points on the surface; where a is the radius of the wire, r is the distance from the centre of the coil. The equation to the surface is

$$r^2 = p^2 \sin\theta,$$

where r is radius to point, θ the angle made with axis, and p is a constant. In commercial instruments of the class considered, the windings, as a rule, are not graded.

Effect of Periodicity on Soft Iron Instruments

The instrument used in the experiments described hereunder was of the ordinary soft iron type with moving armature as used for continuous currents, and probably the errors when using it with alternating current are larger than in an instrument designed for use with alternating current.

A current of 4 amperes was passed through the instrument, and the periodicity varied with the following results:

Periodicity .	12	20	30	40	50
Reading . .	3·95	3·94	3·93	3·90	3·86

Eddies.—The effect of eddies is assumed to obey the same law as that obtained for the series coil of the induction meter, viz.:

$$i_{\text{inst}} = \frac{I}{\sqrt{1 + kn^2}} \quad . \quad . \quad . \quad \text{(i.)}$$

where i_{inst} is proportional to the magnetising component of current. By producing the curve obtained backwards to zero periodicity, I was estimated to be 3·96 for that point, the value of k being calculated from the table above, its mean value being 0·00001756.

To check this value of k, the values of n respectively equal to the frequencies above were substituted in (i.), and the following table obtained:

Periodicity .	12	20	30	40	50
i observed .	3·95	3·94	3·93	3·90	3·86
i calculated .	3·955	3·947	3·929	3·90	3·877

Considering the difficulty in taking readings, the agreement between the calculated and observed results appears to confirm the theory.

Difference in Reading between D.C. and A.C.—In soft iron instruments the force acting at any moment $\propto$ product of field strength produced by current $\times$ field strength in iron.

Since the path of the lines is mostly through air, the former quantity can be taken as proportional to the current; the second portion of the product will depend on the magnetisation curve of the iron.

Generally, low flux densities in the iron are employed, and since in this case B $\propto$ magnetising current, the force on the moving iron will be

$$F \propto i^2,$$

and the mean force obtained with a continuous current will be the same as that of an alternating current with the same R.M.S. value.

If, however, we magnetise the iron so that the magnetisation curve departs from the straight line law, the readings on A.C. and D.C. will generally differ.

It is possible, if the magnetisation curve for the iron is known, to obtain a number corresponding to the product

$$\text{Instantaneous B} \times \text{instantaneous current},$$

i.e. proportional to the instantaneous deflecting force. This force integrated over half a cycle and averaged would give the mean force f_1 due to a given alternating current for a definite position of the iron.

In the same way the same R.M.S. value with continuous current would produce a force f_2 for the same position of the iron, and it is found experimentally that

$$f_2 > f_1.$$

Special Case.—The above remarks will be better understood by considering the following example:

Assume an alternating current of I maximum value and a continuous current C, C being equal to the R.M.S.

value of the alternating current, and assume a magnetisation curve of the iron as below (Fig. 46):

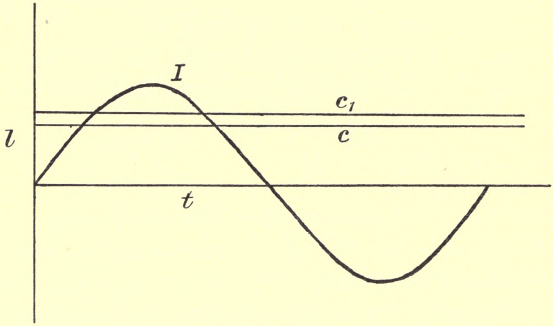


FIG. 45.

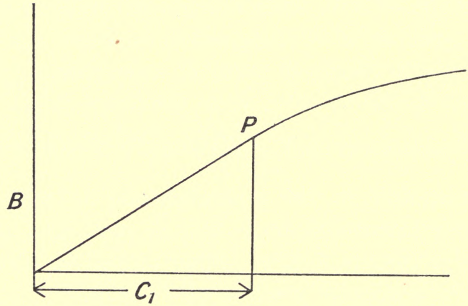


FIG. 46.

The curve is practically straight for the iron and gap in the instrument up to some point P, C_1, the corresponding current being $< I$, but $> C$.

For instantaneous values of the A.C. less than C_1, the instantaneous values of the force will $\propto iB$, *i.e.* as i^2 or $= ki^2$. For values greater than C_1 the force is $< ki^2$. Therefore the total force throughout one half period is

$$< k\int_0^{I/2} i^2 dt,$$

or it is $< k \times$ (R.M.S. current)^2.

In a similar manner we can show that the force due to a direct current $= kC^2$.

Thus the force, and consequently the deflection due to D.C. is $>$ that due to the equivalent A.C.

Wave Form.—It can be shown that if we compare the action of two dissimilar waves of same R.M.S. values with the proviso that the value C_1 in one case is never exceeded, and that this value of the instantaneous current is exceeded in the other case, then it follows that the reading with a flat-topped wave is $>$ that with a peaky wave.

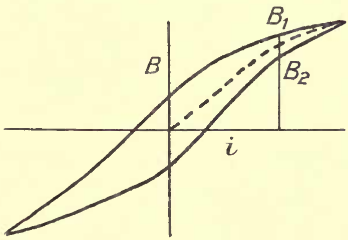


FIG. 47.

Hysteresis.—Fig. 47 represents the usual hysteresis loop sheared over owing to the presence of an air gap.

Corresponding to every value of i we have two possible values of B, *i.e.* B_1 or B_2, according as we consider the ascending or descending limb of the loop.

If as the current increases from 0 to I_{max}, $\frac{di}{dt}$ is always positive, and similarly $\frac{di}{dt}$ is always negative from $+I_{max}$ to $-I_{max}$, *i.e.* so long as the curve is not of the form shown in Fig. 48, the iron will always follow a definite magnetisation curve, and for each half cycle we have two

equal values of i. Thus we can pair the equal values and say that the mean force due to i at P and Q

$$\propto i\left(\frac{B_1+B_2}{2}\right).$$

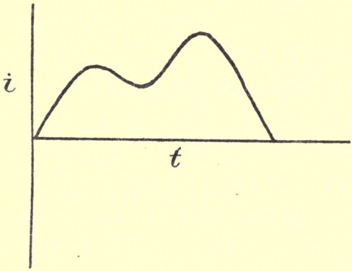


FIG. 48.—Distorted wave.

Plotting $\frac{B_1+B_2}{2}$ and i gives the dotted curve shown in Fig. 47, which at first is practically straight.

The following tests were made at high and low periodicity.

Ammeter.—The one previously tested (p. 145) was run from the Pyke Harris Alternator and a Brush

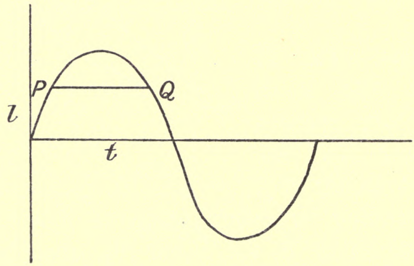


FIG. 49.

Machine, the former of which gives a peaky wave, having the equation

$$e=100 \sin pt-30 \sin 3pt+6 \sin 5pt$$

on open circuit. The current was measured on an ampere balance and kept constant at 4 amperes.

RESULT OF TEST

Periodicity.	Pyke Harris.	Brush Machine Sine Wave.	Calculated.
50	3·81	3·87	3·877
60	3·79	3·84	3·841
70	3·76	3·80	3·801
80	3·72	3·76	3·757
85	3·70	3·72	3·735

Testing in a similar way on low periodicity with a machine giving practically a sine wave gave the following results :

True Current.	Direct Current.	Alternating 12 cycles per second.
1·0	1·00	1·00
1·5	1·50	1·50
2·0	2·01	2·00
2·5	2·52	...
3·0	3·00	2·97
3·5	3·50	...
5·00	5·00	4·90

This test shows clearly that with larger currents the readings on alternating are distinctly less than on direct current even at such low periodicities. The low

periodicity of course eliminates all eddy current effects. The calculated reading for four amperes at zero periodicity was 3·96, and the observed reading was 3·950 at 12 cycles per second.

Voltmeters.—In addition to the errors due to eddy and other effects already discussed in the case of ammeters, voltmeters have another defect, due to increase of impedance at the higher periodicities, since the current i depends on

$$i = \frac{E}{\text{Impedance}}.$$

For instance, a moving iron voltmeter having a resistance of 60 ohms and $L = 0·0775$ Henries gives the following calculated results :

Periodicity.	Impedance.
0	60
10	60·2
20	60·7
30	61·8
40	63
50	64·75

On higher ranges with a series resistance of 588 ohms, the reactance at 50 cycles caused a change of 0·1 per cent, so that at low ranges there is about 8 per cent change, and on high ranges 0·1 per cent.

The following table gives the change in impedance calculated, taking impedance at 40 periods as a standard :

Frequency.	Impedance.	Change.	% Change.
D.C.	60	– 3·0	– 4·76
10	60·2	– 2·8	– 4·45
20	60·7	– 2·3	– 3·56
30	61·8	– 1·2	– 1·91
40	63	0	0
50	64·75	+ 1·75	+ 2·78

The change in alternating readings appears to be fully accounted for by the change in impedance which shows the small effect of eddies in this particular instrument. This is also confirmed on the higher range where the readings between 50 cycles and the lowest obtainable agree with the direct current readings.

If Higher Harmonics are present and the E.M.F. is of the form

$$e = A_1 \sin (pt + a) + 3A_3 \sin 3(pt + a) + \text{ etc.},$$

the presence of the harmonics will tend to reduce the readings, though only to a small extent.

This is obviously the case since the R.M.S. value of e, viz.

$$\sqrt{\frac{A_1{}^2 + A_3{}^2 + \text{ etc.}}{2}},$$

will only exceed $\frac{A_1}{\sqrt{2}}$ by one half of 1 per cent, supposing that A_3 was as great as $\frac{1}{10}A_1$.

CHAPTER V

ELECTROSTATIC ELECTROMETER AND VOLTMETER

THE quadrant electrometer devised by Lord Kelvin has been somewhat modified in the Dolezalek pattern. In this case the quadrants remain as previously, but they are supported on amber insulators, and the needle is made of silvered paper and suspended by means of a quartz fibre. Sometimes the quartz fibre is rendered conducting by means of a hygroscopic solution, so that the needle can be charged from the mains or other source of supply. In some cases phosphor bronze strip is employed. It is found to retain its charge approximately constant for considerable periods, although immediately after charging the instrument takes some little time to adjust for testing purposes. The original Kelvin electrometers are described in *Electrostatics and Magnetism*, p. 261, "Report on Electrometers and Electrostatic Measurements."

It is shown in text-books that the force tending to move the quadrants is proportional to

$$(V_2 - V_1)\left\{V_0 - \frac{V_1 + V_2}{2}\right\}.$$

As a rule V_0 is very high compared with $V_1 + V_2$, so

that we may write for an instrument used heterostatically

$$(V_2 - V_1)V_0 = k\theta,$$

where k is a constant and θ the angle of deflection, V_1, V_2 are the potentials on the quadrants.

The above expression is easily arrived at by regarding the electrometer as a condenser.

If $r\delta\theta$ represents distance moved by the needle, where r is some mean radius, $Fr\delta\theta$ represents work done. But in moving over this distance, the capacity is increased by

$$\frac{br\delta\theta}{8\pi}(V_0 - V_1)^2$$

on the one set of quadrants, and diminished by

$$\frac{br\delta\theta}{8\pi}(V_0 - V_2)^2$$

on the other set.

$$\therefore \quad Fr\delta\theta = \frac{br\delta\theta}{8\pi}\{V_0^2 - 2V_0V_1 + V_1^2 - V_0^2 + 2V_0V_2 + V_2^2\}$$

or

$$F = \frac{k}{4\pi}(V_2 - V_1)\left\{V_0 - \frac{V_1 + V_2}{2}\right\}.$$

This force multiplied by r is then equated to the torsional torque to obtain the deflection. b is supposed to be some breadth. The energy stored in a condenser is taken as $\frac{1}{2}KV^2$ where K is the capacity.

We see, therefore, that the deflection in such a case is proportional to the difference of potentials between quadrants so long as the needle is at a constant potential.

If the needle and one set of quadrants are connected

together, the instrument is said to be used idiostatically. In that case we have

$$\theta = k(V_1 - V_2)^2,$$

and the deflection is proportional to the square of the potential difference between the quadrants. This is made use of in the multicellular electrostatic voltmeter and in Lord Kelvin's high tension voltmeter. In the latter case the restoring couple was gravity.

We see then that if an alternating voltage be applied to the terminals of a multicellular voltmeter, the mean torque will measure

$$\frac{1}{T}\int_0^T V^2 dt.$$

Consequently by graduating the scale parabolically the readings are proportional to the R.M.S. values, or

$$\sqrt{\frac{1}{T}\int_0^T V^2 dt},$$

where V is the instantaneous voltage, T the periodic time. We see that the readings are independent of frequency or wave form.

The disadvantage of a multicellular voltmeter is the comparatively restricted range, and the fact that the needle is not quite dead beat. This is further referred to in the chapter on Electrostatic Wattmeter, p. 153. In order to obtain a sufficient torque, a number of needles and cells are used, all the needles being mounted symmetrically on one spindle. The fact that these instruments can be used for measuring continuous or alternating volts is a great advantage.

The Electrostatic Wattmeter

The use of the quadrant electrometer as a wattmeter seems to have been originally suggested by Professor Ayrton and Fitzgerald independently, and a paper by J. Swinburne, "Electrometer as a Wattmeter" (*Phil. Mag.* vol. xxxi. p. 504), was published in 1891. It was further investigated for this purpose by Professor E. Wilson, "The Kelvin Quadrant Electrometer as a Wattmeter and Voltmeter" (*Proceedings, R.S.* vol. lxii. p. 356, 1898). In the National Physical Laboratory the electrostatic system has been used for over six years for very accurate measurements of power (see *Jour. I.E.E.*, "The Use of the Electrostatic System for the Measurement of Power," by Messrs. Paterson, Rayner, and Kinnes). Since the accuracy attainable is greater than 5 parts in 10,000, a brief description may not be out of place. The papers by Addenbrooke, Orlich and Schultze might also be consulted.

The advantages claimed for the electrostatic system are :

(1) Great accuracy and independence of frequency or wave form.

(2) The great range.

In the ordinary dynamic type of wattmeter, it becomes increasingly difficult to take into account the distribution of current in the heavy conductors necessary for carrying large currents (this is referred to by Lord Kelvin—see *Math. Papers*, vol. v. p. 589). With the electrostatic system currents up to 3000 amperes are easily measured. Also the electrometer is easily used for 3 phase work and eddy currents are entirely absent.

The instruments are all of the direct deflection type. The disadvantages are want of portability, the smallness of the torque, and slow motion of needle. The resistance for carrying current in this case was water cooled.

In order to enable the wattmeter to be calibrated

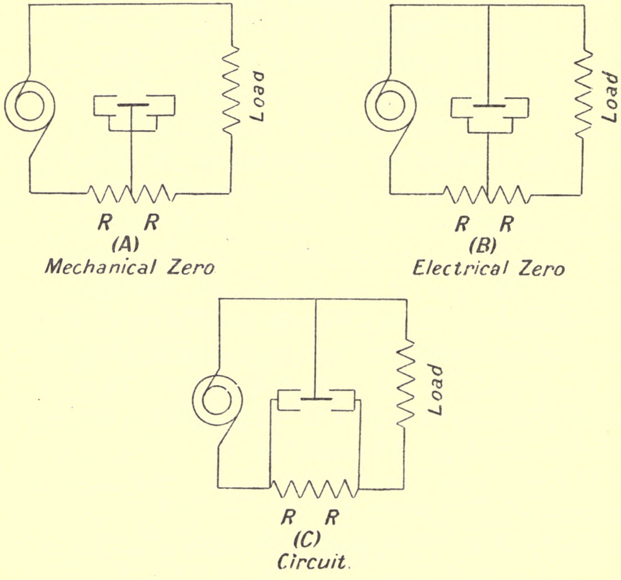


FIG. 50.

on alternating current, an electrostatic voltmeter was arranged with a scale distant 4 metres from the concave mirror, so that volts could be read accurately to 1 part in 10,000. The voltmeter was of the Kelvin multicellular type, and two are used. The method of calibration was to read the volts, say, 100 on a 200 ohm resistance, in which case the watts would be exactly 50.

The wattmeter consisted of a quadrant electrometer with a specially made needle of copper aluminium alloy of very great strength. The quadrants being only 2 millimetres apart, a very accurately made needle was requisite. The great accuracy required made deflections as great as 45° desirable, and the needle gave a straight line law to within 2 parts in 1000. When calibrated, however, an absolutely even scale was not essential.

The suspended system, including the needle, has a periodic time of 8 seconds, and when swinging between quadrants, it came to rest in $1\frac{1}{2}$ oscillations, the periodic time being 16 seconds.

The method of using the instrument will be understood from Fig. 50. For mechanical zero the needle and both quadrants are at the same potential, and for electrical zero the quadrants are at the same potential and needle at a different potential. As a rule these zeros should almost coincide.

If R and R be the resistances in the main circuit, and W the watts expended in L, then it can be shown (see Russell, *Alternating Currents*, vol. i. p. 194) that

$$W = \frac{2k\theta}{R} - I^2R$$

where I^2R are the watts dissipated in R.

Owing to the fact that the constant of the wattmeter varies slightly from point to point on its scale, perfectly even scale divisions cannot be obtained.

The I^2R watts have, of course, to be deducted from the watts measured to obtain the watts W dissipated in L. The method of calibration was as follows. Between A and B there were 200 ohms. The alternator being adjusted so that the electrostatic voltmeter reads 100

volts, exactly 100 watts are dissipated between A and B. By varying the steps, 20 in number of 0·1 ohm on either side of A, the potential on the quadrants can be varied. The two coils with plugs on either side of A are intended for doubling the voltage on the quadrants, when this is

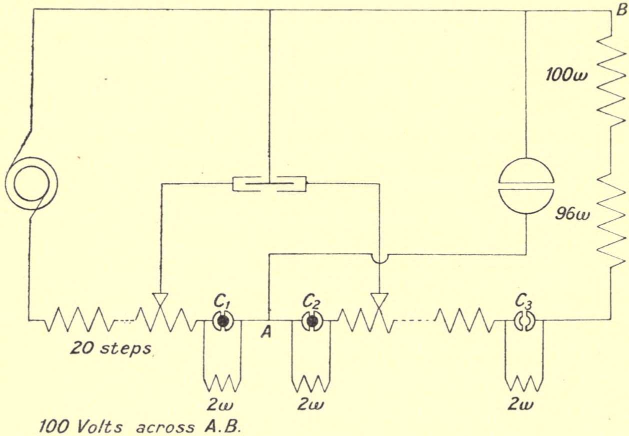


FIG. 51.—Calibration of Electrostatic Wattmeter.

required. C_3 must be short circuited when the others C_1 and C_2 are in use.

The constant then is taken as

$$\frac{E \times I \times R}{D}$$

where E is the voltage between A and B,
I ,, current flowing,
R ,, resistance between quadrants,
D ,, deflection from the electrical zero.

When calibrated the instrument may be used to calibrate

other wattmeters, or make 3 phase measurements of power by the two-wattmeter method.

The electrostatic voltmeter was calibrated by applying a known continuous voltage to it by means of a potential divider consisting of series resistances so arranged that steps of half a volt at a time may be applied from 0 to 250 volts. The current causing this volt drop was $\frac{1}{15}$ of an ampere, and this current was passed through a resistance such that the drop in voltage was balanced against that of a Weston cell, so that the correct steps are obtained in the divider.

The shape of the scale is determined when a concave mirror is used from the equation,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{a\cos\frac{\theta}{2}},$$

where u is the distance of the light source, v the image of a vertical line at a distance L from the mirror M, the angle $\theta=$LMI. A Nernst lamp was used as a source of light.

Some trouble was found with the "creeping" of the deflection owing to torsion in the suspending fibre of the instrument, which necessitated care in marking scale, etc. A mica window bent into a large radius was used, and the image focussed with this in position. Plain glass is useless for such large deflections, although for small deflections it is the most suitable.

The reason why the wattmeter measures the power is at once obvious from the equation for the torque of a quadrant electrometer used heterostatically, viz.

$$T=k(V_2-V_3)\{V_1-\tfrac{1}{2}(V_2+V_3)\},$$

V_1 being the instantaneous potential at B,
V_2 " " " " C,
V_3 " " " " D,

$V_2 - V_3$ being put equal to 2IR in above.

The electrostatic voltmeter is used idiostatically, and its deflections $\propto V^2$.

CHAPTER VI

HOT WIRE INSTRUMENTS

INSTRUMENTS which depend on the heating of a wire by the current are of two distinct classes, viz. those in which the current acts directly, the heating elongating the wire, and those in which the heating causes a rise in temperature, which again acts on a thermo-junction. To the former class belong all hot wire ammeters and voltmeters, to the latter such instruments as Duddell's thermo-galvanometer. Professor Boys's radio-micrometer for measuring the intensity of heat radiations consisted of a coil formed by a thermo-junction suspended between the poles of a powerful magnet, the rays of heat, striking a blackened copper disc attached to the junction, heated it, and a current circulated through the junction, causing a deflection (see Fig. 52).

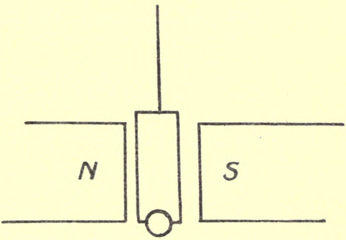


FIG. 52.—Boys's Radio-micrometer.

The earliest instrument of this type used in electrical work was Cardew's voltmeter, which consisted of about 12 feet of a platinum

silver wire taking 10 volts per foot. This wire was enclosed in a tube of compound material, so that its expansion was equal to that of the wire. There were

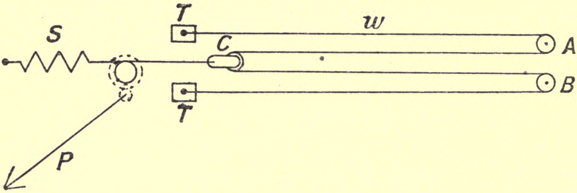


FIG. 53.—Diagram of Cardew's Voltmeter.

w, Wire. A, B, Fixed pulleys.
C, Loose pulley. S, Spring.
P, Pointer.

four lengths of wire passing over pulleys, so that the instrument with dial, etc., was nearly 4 feet long.

Equation for Wire heated by Electric Current.—

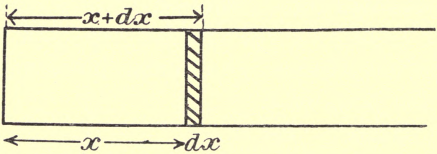


FIG. 54.—Flow of heat in wire.

It is clear that the rate of gain of heat by the element contained by the sections x and $x+dx$ is

$$kA\frac{\partial^2\theta}{\partial x^2}dx,$$

where k is the thermal conductivity for heat, A the area of cross section, θ the temperature.

At the surface of the element, if the perimeter is $2\pi r$, the heat lost, taken proportional to the difference of temperature $\theta - \theta_0$ between it and the air, is

$$\epsilon(\theta - \theta_0)2\pi r dx,$$

where ϵ is its emissivity and r its radius.

The heat gained by the element due to the current is

$$\frac{i^2 \rho dx}{A},$$

where i is the current strength, ρ is the specific resistance.

The total then is now

$$\left(kA\frac{\partial^2\theta}{\partial x^2} - \epsilon(\theta - \theta_0)2\pi r + \frac{i^2\rho}{A}\right)dx.$$

Since the elementary mass is rising in temperature at the rate $\frac{\partial\theta}{\partial t}$, we must have the relation

$$A \,.\, s \,.\, d \,.\, \frac{\partial\theta}{\partial t}dx = \left(kA\frac{\partial^2\theta}{\partial x^2} - \epsilon(\theta - \theta_0)2\pi r + \frac{i^2\rho}{A}\right)dx,$$

where s is the specific heat, d the density ; consequently the equation is

$$\frac{\partial\theta}{\partial t} = \frac{k}{s \,.\, d}\frac{\partial^2\theta}{\partial x^2} - \frac{2\pi r\epsilon(\theta - \theta_0)}{A \,.\, s \,.\, d} + \frac{i^2\rho}{A^2 \,.\, s \,.\, d}\,.$$

When the temperature has ceased to rise and become steady in the wire, since $\frac{\partial\theta}{\partial t} = 0$ and $\frac{\partial^2\theta}{\partial x^2} = 0$, then we have

$$\frac{i^2\rho}{s \,.\, d \,.\, A^2} = \frac{2\pi r(\theta - \theta_0)\epsilon}{A \,.\, s \,.\, d}\,.$$

Putting $A = \pi r^2$, and cancelling, we have

$$i = \sqrt{\frac{2\pi^2 r^3(\theta - \theta_0)\epsilon}{\rho}}.$$

Assuming that the emissivity is constant (which is not, strictly speaking, correct) for all temperatures, let

$$k_1 = \sqrt{\frac{2\pi^2\epsilon}{\rho}},$$

then $$i = k_1 r^{3/2} . (\theta - \theta_0)^{\frac{1}{2}}.$$

We see that the rise of temperature $\propto$ square of current multiplied by the three halved power of the radius of the wire.

Let l_0 be the length of the wire at temperature θ_0 and l its length at temperature θ, then, if a is the linear coefficient of expansion for the wire,

$$l = l_0\{1 + a(\theta - \theta_0)\}$$

or $$\frac{l - l_0}{al_0} = \theta - \theta_0,$$

but $l - l_0 = \Delta l$, the increase in length of the wire, so that

$$\theta - \theta_0 = \frac{\Delta l}{al_0}.$$

Substituting in our formula we have

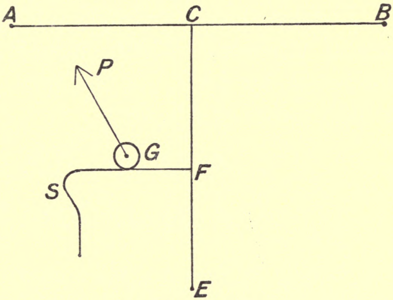
$$i = k_1 r^{3/2}\left(\frac{\Delta l}{al_0}\right)^{\frac{1}{2}}.$$

Since a is a constant for the material and l_0 a given initial length, we might write this

$$i = K . r^{3/2} . (\Delta l)^{\frac{1}{2}}.$$

We see, therefore, that the increment in length depends

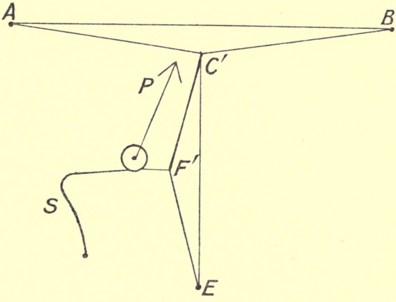
on square of current when the radius is constant. Such instruments, therefore, involving a square law possess the advantage of measuring the R.M.S. value of any alternating current, and the disadvantage of unequal scale division owing to the same cause.



Undeflected.

Fig. 55.

In the Cardew voltmeter the expansion of the platinum wire was multiplied by means of gear wheels, as can be seen from figure 53.



Deflected

Fig. 56.

Hartmann and Braun hot wire instrument.

Hartmann and Braun ammeters and voltmeters again use a multiplying arrangement which, besides being very ingenious, multiplies the deflection due to the expansion very greatly. In this arrangement AB is the wire carrying the current, C^1 is a point to which the wire EC^1 is attached, and this again is fixed at E. To a point F another wire passes over a pulley to which the pointer is attached and finally fixed to a spring.

When AB sags a distance s, CE moves out a distance s', being pulled out by the spring, consequently the pointer moves through a distance proportional to s'.

Theory.—Let $2L$ = length of wire originally in horizontal position. When current passes, its length is increased by $2\Delta L$. If the wire E is attached to the middle point of AB, then

$$s^2 + L^2 = (L + \Delta L)^2,$$

$$s^2 + L^2 = L^2 + 2\Delta L \,.\, L + \Delta L^2.$$

Neglecting ΔL^2 as a small quantity,

$$s = \sqrt{2\Delta L \,.\, L}.$$

Proceeding now to E, let its length $= 2l$, then

$$l^2 - s'^2 = \left(l - \frac{s}{2}\right)^2,$$

$$l^2 - s'^2 = l^2 - ls + \frac{s^2}{4},$$

$$\therefore \quad s' = \sqrt{ls - \frac{s^2}{4}}.$$

And $\qquad s = \sqrt{2\Delta L \,.\, L}$ approximately,

since ΔL is a small quantity $s' = \sqrt{ls}$. Now $s = \sqrt{2\Delta L \,.\, L}$, and ΔL as shown above $\propto i^2$,

$$\therefore \quad s \propto i.$$

Hence $$s' \propto i^{\frac{1}{2}}$$

or $$s' \propto l^{\frac{1}{2}} \,.\, i^{\frac{1}{2}}.$$

Example.—Suppose $s=\frac{1}{100}$ cm., $l=5$ cms.,

then $$s'=\sqrt{\frac{5}{100}}\text{ cm.}$$

or $$s'=\frac{1}{4\cdot 4}\text{ cms.}$$

$$\therefore \quad s'/s=\frac{100}{4\cdot 4}\doteq 22\cdot 7,$$

or a multiplication of motion of 22·7 times. By means of the pulley and pointer it is still further increased.

We see from above that the scale divisions, instead of varying with the square of current, as in Cardew's instrument, will vary with the square root of current. If R is the reading of the instrument, we have

$$R=kI^{\frac{1}{2}},$$

$$dR=\tfrac{1}{2}kI^{-\frac{1}{2}}dI,$$

$$\therefore \quad \frac{dR}{R}=\frac{dI}{2I},$$

or the error in reading $\propto \frac{1}{I}$. In the Cardew instrument

$$R=kI^2,$$

$$dR=2kIdI,$$

$$\therefore \quad \frac{dR}{R}=\frac{2dI}{I},$$

so that the error in reading a small change of current for the Hartmann and Braun instrument is only $\frac{1}{4}$th that of the instrument of the Cardew type. In both instruments this error diminishes as C increases proportionally.

Hot Wire Ammeter.—Locus of moving point forms a confocal system. Let $2l$ = length of expanding wire. Take its centre as origin, and let P be any point on the expanded wire. For any given temperature we see that

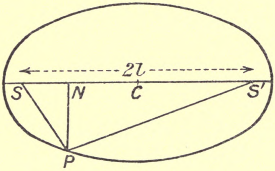


FIG. 57.—Moving point in heated wire.

$$SP + S'P = 2l(1 + a\theta).$$

But
$$e = \frac{l}{l(1 + a\theta)},$$

where e is the eccentricity.

Hence, referred to the centre as origin, our equation for locus of P is

$$\frac{x^2}{l^2(1 + a\theta)^2} + \frac{y^2}{l^2(1 + a\theta)^2} \times \frac{1}{\left\{1 - \frac{1}{(1 + a\theta)^2}\right\}} = 1,$$

evidently an ellipse with foci at SS′, the ends of the original wire.

This equation might be written, assuming $a\theta$ a small quantity,

$$\frac{x^2}{l^2 + 2l^2 a\theta} + \frac{y^2}{l^2 2a\theta} = 1.$$

If we write $l^2 = a^2$ and $2l^2 a\theta = \lambda$, we have

$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{\lambda} = 1,$$

and since λ is always a positive quantity, we see that this equation is that of a system of confocal ellipses. As λ becomes greater the system becomes approximately circles, and as λ gets very small it approaches a line ellipse, *i.e.* the wire itself is one of the confocals.

Since we see that the locus of P is always an ellipse, obviously the maximum sag is obtained by fixing the controlling wire to the centre of the expanding wire.

Generally speaking, the controlling wire is not attached to the centre of the expanding wire, and we may regard the matter from another point of view.

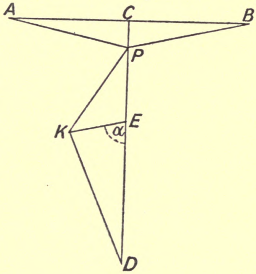


FIG. 58.—Hot wire ammeter multiplying device.

$PK = CE = l_1$ $PE = l_1 - S = m$
$CP = S$ $CE + ED = 2l$
$KED = a$ $KE = x$
$ED = l_2 = KD$

From the triangle PKE

$$l_1^2 = x^2 + m^2 + 2xm \cos a,$$

but $$\frac{x}{2l_2} = \cos a,$$

$$\therefore \quad l_1^2 = x^2 + m^2 + \frac{x^2 m}{l^2},$$

$$x^2\left(1 + \frac{m}{l_2}\right) = l_1^2 - m^2,$$

$$x^2 = \frac{l_2(l_1^2 - m^2)}{l_2 + m},$$

and substituting $$m = l_1 - S,$$

$$= \frac{l_2(l_1^2 - (l_1 - S)^2)}{l_2 + m},$$

$$=\frac{l_2(2l_1S - S^2)}{l_2 + m},$$

$$=\left(\frac{S}{l_1 + l_2 - S}\right)(2l_1l_2 - l_2S),$$

if

$$l_1 = l + Z,$$

$$l_2 = l - Z,$$

$$x^2 = \frac{S}{(l_1 + l_2 - S)}\{2(l^2 - Z^2) - (l - Z)S\},$$

$$= \frac{S}{(2l - S)}(2l^2 - lS - 2Z^2 + ZS).$$

The max. value of x^2 is when $(2Z^2 - ZS)$ is a min.,

$$\frac{d}{dz}(x)^2 = 2x\frac{dx}{dz} = 2x\left(\frac{S}{2l - S}\right)(-4Z + S).$$

When $Z = \frac{S}{4}$ we have a max. or min. Hence

$$l_1 = l + S/4,$$

$$l_2 = l - S/4.$$

Since S is a function of the current to obtain max. value of EK, we should have to vary the position of E for each value of current. Hence we can either make

$$l_1 = l + S/4, \text{ when S say}$$

corresponding to max. current, or more simply make

$$l_1 = l.$$

Several Sections of Wire in Parallel.

As shown later, if the current distribution is not uniform, the heat generated is greater than with uniform distribution.

In some ammeters for high ranges the device shown in Fig. 59 is employed.

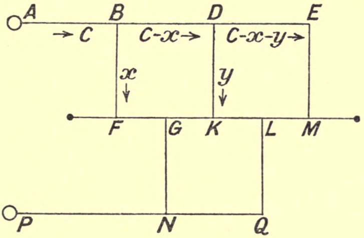


Fig. 59.—Hot wire ammeter. Several sections of wire in parallel.

PNQ, ABDE } thick copper leads of negligible resistance.

FGKLM, working wire.

BF, DK, EM, GM, LQ, thick silver strip.

Let res. of strips BF, DK, etc., assumed all equal, be R.

Let res. of FG, GK, KL, LM be each r.

By symmetry

current through BF = current through EM,

$$x = \mathrm{C} - x - y,$$

or

$$\mathrm{C} = 2x + y \quad . \quad . \quad . \quad (1)$$

Going round circuit BFGKD,

$$x\mathrm{R} + rx - \frac{ry}{2} - \mathrm{R}y = 0,$$

$$2x(\mathrm{R} + r) - y(r + 2\mathrm{R}) = 0 \quad . \quad . \quad (2)$$

Solving for x

$$x=\frac{\begin{vmatrix} C & 1 \\ 0 & -(r+2R) \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 2(R+r) & -(r+2R) \end{vmatrix}},$$

$$=\frac{-C(r+2R)}{-2r-4R-2R-2r},$$

$$=\frac{C(r+2R)}{(4r+6R)},$$

and $$y=\frac{4R+2r}{4r+6R}C.$$

If R be small compared with r

$$x=\frac{Cr}{4r}=C/4.$$

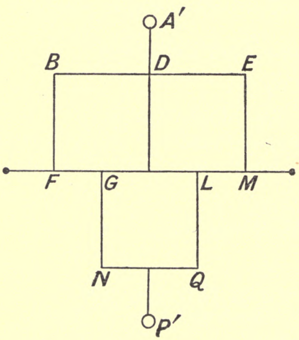


Fig. 60.—Hot wire ammeter. Several sections of wire in parallel; alternative connections.

If the resistances of ABDE, PNQ be considered, the expression becomes more complicated, and the results will be different according as we assume current to be led in at A and P or as in figure 60, which being more symmetrical should give more uniform distribution.

Effect of Unequal Current Distribution.—

Let C = total current.

Assume conductor divided into n equal elements in

parallel, each of resistance r. Then current per element $=$ $\frac{C}{n} = \iota$ (say) for uniform distribution.

$$\text{Watts} = C^2\frac{r}{n} = \frac{n^2\iota^2 r}{n} = n\iota^2 r.$$

Let actual distribution be

$$\iota + x_1, \quad \iota + x_2, \quad . \; . \; . \quad \iota + x_n,$$

where x_1, x_2 . . . may be $+$ve or $-$ve.

$$\begin{aligned}\text{Watts} &= \overline{\Sigma(\iota + x)^2 r,} \\ &= r(\overline{\iota + x_1}^2 + \overline{\iota + x_2}^2 + \;.\;.\;.\; + \overline{\iota + x_n}^2), \\ &= nr\iota^2 + 2r\iota(x_1 + x_2 + \;.\;.\;.\; + x_n), \\ &\quad + x_1^2 r + x_2^2 r + \;.\;.\;.\; + x_n^2 r.\end{aligned}$$

But $$x_1 + x_2 + \;.\;.\;.\; + x_n = 0.$$

Hence $\text{Watts} = nr\iota^2 + r(x_1^2 + x_2^2 + \;.\;.\;.\; + x_n^2)$,

which is greater than with uniform distribution, and which is in accordance with Kirchoff's laws.

Hot Wire Wattmeter

Interesting instruments have been devised by Messrs. M. B. Field and J. T. Irwin (*vide Proceedings I.E.E.*, " Hot Wire Wattmeters and Oscillographs, vol. xxxix. p. 617).

The principle involved depends on the fact that if xy represents the instantaneous power in a circuit, then

$$4xy = (x + y)^2 - (x - y)^2,$$

or the power is proportional to the square of the sum of volts and drop in resistance minus the square of the difference of volts and drop in a resistance.

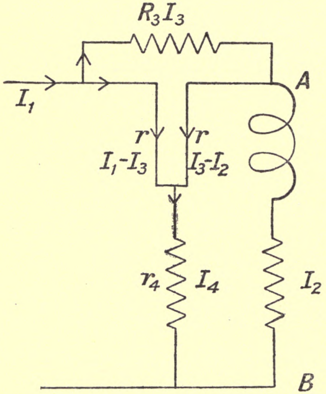


FIG. 61.

Let R_3 be a resistance in the main circuit, r_4 a resistance in series with the strips *a* and *b*, carrying current $(I_1 - I_3)$ and $(I_3 - I_2)$ respectively. Then if r is the resistance of each strip

$$I_3R_3 + (I_3 - I_2)r = r(I_4 - I_3 + I_2),$$

since $\quad I_1 = I_2 + I_4$

or $$r\{2(I_2 - I_3) + I_4\} = I_3R_3 \quad . \quad . \quad . \quad \text{(i.)}$$

The difference in the rate of heating is

$$0{\cdot}24 \times r(I_4 - I_3 + I_2)^2 - r(I_3 - I_2)^2 = x.$$

From equation (i.) above it can be shown that

$$x = \frac{{\cdot}24 r I_4 I_3 R_3}{r}, \text{ or } x = {\cdot}24 I_4 I_3 R_3 \text{ calories per sec.}$$

The current

$$I_4 = \frac{1}{r_4}\{V - r(I_3 - I_2)\},$$

and when $(I_3 - I_2)r$ is small compared with V, the voltage across the load, and I_3 approximately equal to I_2, we see that the reading of the instrument

$$\propto I_2V,$$

or the watts in the circuit AB.

This is true for instantaneous values, hence true for mean.

To obtain the maximum difference of heating, the arrangement should be adopted that the current through the strips when the pressure current is taken off should be equal to the pressure current when the main current is zero. In this case, at maximum load with power factor unity, no current will flow in one of the strips.

Tests of a hot wire wattmeter constructed by Mr. Irwin on a similar principle to that used in his hot wire oscillograph gave practically straight lines from 0 to 700 watts, when the deflection was plotted against volt amperes.

On an inductive load the deflection is proportional to watts, plus a constant.

So far the hot wire wattmeter has not apparently been much used, but the principle involved is interesting, and may lead to developments later.

Thermo-Galvanometers.—These instruments of Duddell pattern and the thermo-ammeter depend on the principle that a current to be measured is passed directly through a heater consisting of a resistance coil. This acts upon one junction of a thermo-junction and raises its temperature; consequently a current will flow on the thermo-junction circuit (see Fig. 52). This current flowing through a wire in a magnetic field causes a deflection of a spot of light, or pointer. These instruments are of importance at present to measure currents in wave meters, wireless circuits, X-ray tubes and in telephony.

It is shown in text-books (*vide* J. J. Thomson, *Electricity and Magnetism*, p. 496) that if P_1 is the Peltier effect at a cold junction at temperature absolute

T_1 and P_2, the same effect at the hot junction at temperature T_2, then since it obeys the second law of thermodynamics

$$\frac{P_2}{T_2}=\frac{P_1}{T_1}=\frac{\text{work done by unit of electricity}}{T_2-T_1}.$$

Since the work done is E, we have

$$E=\frac{T_2-T_1}{T_1}P_1.$$

Leaving aside then Thomson effects, and assuming only Peltier effect as the only reversible one at the junction,

$$E=k(T_2-T_1),$$

or E depends only on difference of temperature.

Generally speaking E is measured by

$$\int_{T_1}^{T_2} Q\,dt,$$

where Q is the thermo-electric power of the junctions considered.

Now if T_2-T_1 depends on I^2R, we have, writing θ for the difference of temperature,

$$E=k_1\theta=k_2I^2R,$$

since i the current in the thermo-circuit $=\frac{E}{r}$, r being the galvanometer resistance

$$i=\frac{k_2I^2R}{r}.$$

Hence if i was measured directly, then

$$I \propto \sqrt{\theta},$$

or the scale would be divided accordingly.

With a resistance of 1000 ohms in the heater, 100 microamperes give a deflection of 250 mms. If the resistance of the heater was reduced, the current would have to be increased, so that I^2R was a constant quantity, in order that the same deflection be obtained. Clearly the instrument is independent of temperature changes, wave form and periodicity, and so can be calibrated with direct current and then used on any other circuit carrying oscillating or alternating currents.

The Joule Radiometer.—An interesting instrument of this type has been devised by Mr. F. W. Jordan of the South-Western Polytechnic. It depends for its action on convection currents of air acting on a light vane attached to a quartz fibre. A mirror and scale are necessary (*vide Proceedings Phys. Soc. of London*, vol. xxv. Part I. Dec. 15, 1912, "An Improved Joule Radiometer and its Applications").

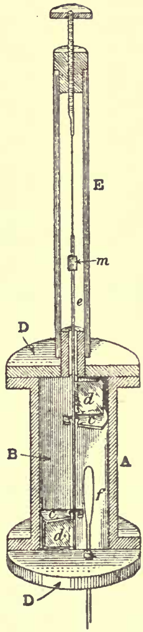


FIG. 62.—Jordan's Radiometer.

A, Brass case.
B, Copper plate forming partition.
D, D, Base and cover.
E, Brass tube.
c, c, Sector-shaped copper discs.
d, d, Mica vanes.
e, Glass stem.
f, Heater.
m, Mirror.

The heater consisted in one form of instrument of

Eureka wire through which the current to be measured was passed. The vanes, two in number, were of mica fixed to a glass stem, and this in turn attached to a quartz fibre. The air inside the instrument was practically sealed and the heat dispersed by means of the brass case. The deflection at a meter was 0·52 m/mms. per microwatt. The smallest current detectable was 5×10^{-5} ampere.

Since the heated air rising to strike one vane and the cold air descending to strike the other might be supposed to have velocities v, this velocity varies as $\sqrt{T - T_0}$. The force with which it strikes the vane then is kv^2 or

$$k(T - T_0),$$

where k is some constant.

But $T - T_0 \propto I^2R$, the rate of heating in the Eureka strip or wire. Hence the deflection

$$\theta \propto I^2R.$$

Mr. Jordan appears to have made a great improvement, particularly by using a thick brass conductor to carry off heat instead of trying to shield the instrument with non-conductors.

This instrument can also be used to measure radiant heat.

For complete theory the original paper must be consulted.

The thermo-ammeter devised by Professor Fleming to measure the R.M.S. value of currents in wireless telegraphy simply consists of a heater composed of several wires with a thermo-junction of fine iron and Eureka connected to the central wire. The terminals of the thermo-junction are connected to a unipivot ammeter.

CHAPTER VII

DYNAMOMETER INSTRUMENTS

THESE instruments all work by attraction or repulsion between two or more coils carrying currents.

In the current dynamometer and wattmeter of Siemens's type, the movable coil is brought to the zero position by means of a torsion head and index. The wattmeter is considered in the dynamometer wattmeter section, and as regards the current instrument, since the same current circulates in both coils, the attraction depends on I^2.

The basis of all the theory of such instruments as dynamometers or current balances depends upon the mutual electrokinetic energy of the coils considered. If we write, as is usual in text-books, T = electrokinetic energy, then

$$\frac{\partial T}{\partial \phi} = \text{turning couple},$$

or

$$k\theta = \frac{\partial T}{\partial \phi},$$

where ϕ is the angle between the axis of the coils in question.

If, as in the torsion dynamometer, the coils are in the

zero position, ϕ is then 90°. The determination of the electrokinetic energy for the action of the two magnetic shells involves applications of Zonal Surface Harmonics, and for this standard treatises must be consulted.

In the case of two coils of radii a_1, a_2 mutually at right angles carrying the same current with centres coincident, each coil of n turns and both carrying a current I, the expression for the torque is

$$\frac{2\pi^2 I^2 a_2^2 n^2}{a_1} = k\theta$$

where θ is the angle of torsion.

Consequently $\quad I = A\sqrt{\theta}$
where A is a constant.

These instruments, especially when arranged with several windings, have a great range, and the chief objection to them is that they are not dead beat, and the square root of the reading has to be taken. As a rule the movable coil has only one turn; the fixed coil may have several turns.

Since the periodic time of the movable coil is large (20 seconds), alternating currents at ordinary frequency give steady deflections, and the instrument measures

$$\frac{1}{T}\int_0^T C^2 dt.$$

The square root of the reading therefore measures $\sqrt{\text{mean}^2}$ value of the current.

The above outline of the theory merely considers two circular currents. The action, however, between any two mutually influencing coils always involves

$$\text{Torque} = I_1 I_2 n_1 n_2 \{A_1 + A_2 + \text{ etc.}\},$$

where A_1, A_2, etc., are coefficients depending on the dimensions and relative position of the coils. For Siemens's dynamometer the law of squares of current is very closely adhered to indeed, and it can easily be tested by plotting logarithms of current and angle of torsion.

In Lord Kelvin's current balances there were generally six coils. Four of these are fixed as AA_1, BB_1; the other two CC were at the ends of a balance arm. The coils CC were therefore acted upon on both sides, and since the current in CC circulates in opposite

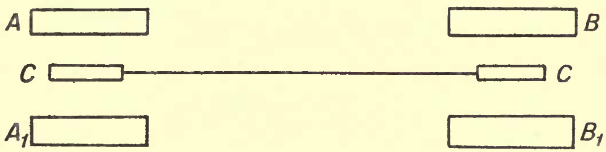


FIG. 63.—Kelvin Ampere Balance.
A, A_1, B, B_1, Fixed coils. C, C, Moving coils.

directions in the two coils, the effect of earth's magnetism is rendered negligible.

In the Kelvin balance the coils are relatively large and close together, and it can be shown that for two fixed coils acting on a third movable one on opposite sides the force is

$$F = \frac{\partial M}{\partial \zeta} = \frac{\pi^2 a_1 a_2}{r^4}\left(1 \,.\, 2 \,.\, 3 \frac{x}{r} + 3 \,.\, 4 \,.\, 5 \frac{x(x^2 - \frac{3}{4}a^2)}{r^5}(\zeta^2 - \tfrac{1}{4}a^2) + \text{ etc.}\right).$$

If a_1 is the radius of the larger coils, a_2 that of the smaller movable one, then, if we take the first term only, and differentiate it, and equate to zero, we find for maximum force then $x = 2a$, and in this way

$$f = 0{\cdot}28 \times 6\pi^2 \frac{a_2^{\,2}}{a_1^{\,2}} \times I^2 \times n_1 n_2.$$

For two coils acting together the force is doubled, and for the four coils quadrupled. Hence in reading the instrument, a table of doubled square roots of readings is used. The coils initially are so arranged as to be in the "sighted" position, and when current passes the forces upset the balance, and it is again restored by sliding a travelling weight along the beam. In using a Kelvin balance the work is somewhat tedious owing to the motion of the beam not being much damped, and while waiting for it to come to rest the current may fluctuate slightly. For the purpose of calibrating other instruments in the laboratory it is, however, almost a *sine qua non* if the laboratory is well equipped.

In instruments of the Siemens dynamometer type, one method of increasing the range is to shunt the

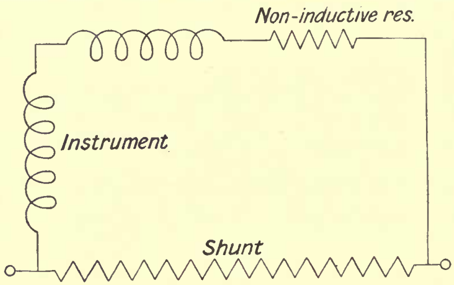


FIG. 64.—Shunted dynamometer ammeter.

instrument, as in Fig. 64. Now if M is the multiplying power of the shunt for continuous currents, M^1 for alternating, then (Drysdale, *Phil. Mag.* vol. xvi. p. 138) it has been shown that

$$M^1 = M\sqrt{1 + \frac{M-1}{M}T^2p^2},$$

where T is the time constant and $p = 2\pi n$.

It is shown (*vide* Ridsdale, "Dynamometer Ammeters and Voltmeters," vol. xlviii. *Jour. I.E.E.* p. 515) that shunting in this way may introduce considerable frequency and temperature errors, unless the time constant of the shunt equals that of the instrument, which is difficult to accomplish in practice.

If, however, the moving coil only of such an instrument be shunted, it is possible to produce an ammeter which will read correctly both on alternating and on continuous circuits.

Ridsdale shows that in this case the percentage error is given by

$$\frac{100pT}{2}\left(\frac{pM}{R}+pT\right),$$

which at 100 cycles gives only 0·01 per cent error, and the temperature error only $\frac{1}{6}$ of 1 per cent. The error due to wave form is also shown to be about $\frac{1}{10000}$ part, so is negligible.

Making an accurate dynamometer voltmeter is a simpler matter than making an ammeter.

It can be shown that in this case the ratio of the voltages on alternating current circuit to produce the same torque as on a continuous circuit is

$$\sqrt{\frac{R^2+p^2(L+M)^2}{R^2}}$$

or $1+\frac{1}{2}p^2T^2$ nearly, where

$$T=\frac{L+M}{R},$$

the time constant of the instrument. At a frequency

of 100 the error is only $\frac{1}{20}$ of 1 per cent, and mutual induction produces much less effect than in ammeters.

In the voltmeter the coils are in series together with a large non-inductive resistance.

For recording purposes the dynamometer voltmeter possesses many advantages, but owing to the shunt volt drop, the ammeter is not so satisfactory as other instruments. For laboratory minor standards these instruments are of importance.

Shunted A.C. Ammeter.—For a shunted alternating current ammeter of this description we have

$$M^1 = M\sqrt{1 + \left(\frac{M-1}{M}\right)^2 T^2 p^2}.$$

For a given instrument and shunt M and T are constant, so that we may write

$$M^1 = M\sqrt{1 + kn^2},$$

where n is the periodicity.

To test this a soft iron instrument was used to increase any error, and it was shunted by means of a piece of Eureka wire which increased its range in the ratio of 3/1. This instrument was then connected in series with a dynamometer ammeter which was known to be unaffected by periodicity. The periodicity was varied between 20 to 50, the reading of the second instrument being kept constant at 9 amperes.

The periodicity error curve of the instrument was practically a straight line over the range, the actual figures being:

[TABLE

True Current.	Periodicity.	Reading of Instrument.	Current through Instrument from Calibration Curve.	Calculated.
9·00	50	2·86	2·93	2·93
"	45	2·88	2·94	2·94
"	40	2·90	2·95	2·953
"	35	2·92	2·96	2·965
"	30	2·94	2·97	2·974
"	25	2·96	2·98	2·981
"	20	2·98	2·99	2·99
"	D.C.	3·00	3·00	...

Hence, we have $M = 3$, and when periodicity is 50

$$M^1 = \frac{9}{2{\cdot}93}.$$

Hence, putting

$$1 + kn^2 = \left(\frac{9}{3 \times 2{\cdot}93}\right)^2,$$

$$1 + kn^2 = 1{\cdot}05,$$

or

$$k = \frac{0{\cdot}05}{50^2}.$$

Employing this value of k, the calculated current through the instrument was obtained as shown in column 5 above. It agrees very well with the observed values in column 4.

The Dynamometer Wattmeter

This instrument generally consists of a fixed coil carrying the main current and a fine wire coil carrying the pressure current. Sometimes, in direct deflection instruments, the coils are arranged astatically to render magnetic effects in their neighbourhood negligible. In the dynamometer pattern the torsion head brings the movable coil perpendicular to the plane of the fixed coil when in the zero position. When a spring control is used, the movable coil is repelled from the fixed coil through an angle to which the watts are supposed proportional and balanced by the torsional couple on the spring.

With continuous currents, the errors in the wattmeter are very small, but since in this case there is no phase angle between volts and current, wattmeters are seldom used except for alternating currents.

In this case several sources of error, all of them comparatively small, creep in. These are as follows:

I. Error due to self-induction of the shunt coil.
II. Error due to capacity of shunt circuit.
III. Error due to mutual induction between the coils.
IV. Error due to eddies generated in brass screws, etc., in the neighbourhood of the coils.
V. Electrostatic attraction between the coils.
VI. Frequency errors.
VII. Wave form errors.
VIII. Effect of earth's magnetism.
IX. Distribution of current in series windings.

The simple theory of the wattmeter is generally arrived at from a vector diagram.

If V is the vector representing the pressure across the shunt coil, then the ohmic volt drop across the shunt coil, plus its series resistance, is

$$ir = \mathrm{V} \cos \lambda.$$

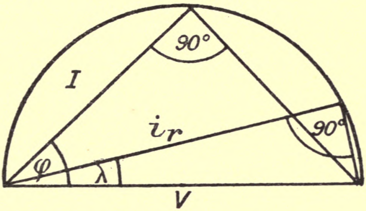


FIG. 65.—Vector diagram for wattmeter.

The true power is the mean value of the harmonically varying quantities I and V with a lag angle ϕ, and as Blakesley has shown is

$$\mathrm{IV} \cos \phi.$$

But the instrument measures the mean torque due to i_1, I_2 in its coils, or is equal to

$$k\mathrm{I}i_1 \cos a.$$

or $$k\mathrm{IV} \cos a \,.\, \cos \lambda,$$

where a is the angle between the volts and current in the coils. When these vectors lie in one plane

$$\cos a = \cos (\phi - \lambda),$$

and we have the expression for the correcting factor when v is the volt drop across the coils

$$\frac{\cos \phi}{\cos \lambda \,.\, \cos(\phi - \lambda)}.$$

As a rule, however, the vectors are not coplanar and

$$a = \phi - \lambda + x.$$

If the angle λ is small, $\cos\lambda = 1$, $\sin\lambda = \lambda$, $\sin(\lambda - x) = \lambda - x$, if we express them in circular measure.

The correcting factor therefore is

$$\frac{\cos\phi}{\cos a \, . \cos\lambda} = \frac{\cos\phi}{\cos(\phi - (\lambda - x))} = \frac{\cos\phi}{\cos\phi + (\lambda - x)\sin\phi}$$

$$= \frac{1}{1 + (\lambda - x)\tan\phi} \text{ approximately.}$$

For sine waves $x = o$, and the expression reduces to the one quoted above. For low power factors the error will be great.

We see also that if the vectors lie out of the plane by an amount $\lambda = x$ it will read correctly on all power factors, if these angles are small and the above equation holds.

For a small phase angle ϕ, we might write the correcting factor

$$1/1 - p\left(\frac{l}{r} - x\right)\tan\phi,$$

where p is the frequency since $\dfrac{pl}{r} = \lambda$ approximately where l and r are the resistance and self-induction of the shunt circuit. If we write for the reactance of the shunt circuit

$$\frac{pl - \dfrac{1}{pk}}{r},$$

we see that if resonance exists, the correcting factor will be

$$\frac{1}{1 - x\tan\phi}.$$

Again, if
$$\frac{1}{1-(\lambda-x)\tan\phi}$$
represents the correcting factor, then if
$$\lambda-x=\frac{1}{\tan\phi}$$
the correcting factor is infinite, and if
$$\lambda-x>\frac{1}{\tan\phi}$$
the deflection is negative.

Generally speaking $\lambda-x$ is the difference of two small quantities, and to make
$$\tan\phi\,.\,(\lambda-x)>\text{ unity}$$
would necessitate a large condenser load.

Dr. Drysdale gives an expression
$$w=w'(1+\mathrm{T}^2p^2)-\mathrm{T}p\mathrm{W}\sin\phi,$$
where w = true watts, w' the reading of instrument, T the time constant, p the frequency, W the apparent watts, and ϕ the angle of lag (*Electrician*, vol. xlvi. p. 774, 1901, and original communication, "The Theory of the Dynamometer Wattmeter," *Jour. I.E.E.*, vol. xliv. p. 255). The above, for well-designed instruments, reduces to the form,
$$w=w'-\mathrm{T}p\mathrm{W}\sin\phi.$$

Effect of Wave Form.—Obviously the true power for a curve composed of various harmonics is
$$w_0=\Sigma v_n\mathrm{C}_n\cos\phi_n.$$
But
$$\phi_n=n(\theta_n-\theta'_n),$$

where θ_n = angle of lag of current in nth harmonic, θ'_n ditto for the volt circuit. The wattmeter reading will be

$$w = \sum \frac{r}{I_n} V_n C_n \cos(\phi - \theta'_n),$$

and since $\frac{r}{I_n}$ where r is the resistance of the volt coil, I_n the impedance for harmonic of order n,

$$\cos \theta'_n = \frac{r}{I_n},$$

$$\therefore \quad w = \Sigma V_n C_n \cos \theta'_n \,.\, \cos(\phi_n - \theta'_n).$$

Expanding, and assuming, that $n\mathrm{T}p$ is small compared with unity, it follows that

$$w = w' - \mathrm{T}p\Sigma n V_n C_n \sin \phi_n,$$

and for sine wave we have,

$$w = w' - n\mathrm{T}pw \sin \phi,$$

which reduces to the expression for above for case when $n = 1$.

Dr. Drysdale also shows that for large values of inductance, the reading is independent of wave form. This follows from the fact that

$$C_n = \frac{V_n}{nx},$$

and $$n\frac{V_n C_n}{x} = \frac{V_n^2}{x}$$

where x is the reactance of the circuit.

Hence $$w = w_0 - \mathrm{T}p\frac{\mathrm{V}^2}{x}.$$

In the case of capacities, however, the error is increased.

By putting $$\mathrm{C}_n = \frac{n\mathrm{V}_n}{x}$$

we obtain $$w_0 = w + \frac{\mathrm{T}p}{x}\Sigma n^2 \mathrm{V}_n{}^2.$$

Unless, therefore, the harmonics are rapidly diminishing, the error will be greatly magnified.

The effect of wave form given in Drysdale's paper is calculated for given form of wave, as follows :

Wave Form.		$w = \frac{w_0 - w'}{\mathrm{W}}$.	
Volts.	Current.	$\phi = 0$.	$\phi = \frac{\pi}{2}$.
Sine . .	Sine . .	0	$-1{\cdot}00 \times 10^{-3}$
Rectangular .	Rectangular .	$0{\cdot}81 \times 10^{-6}n$	$-0{\cdot}62 \times 10^{-3}$
Triangular .	Triangular .	$1{\cdot}2 \times 10^{-6}$	$-{\cdot}96 \times 10^{-3}$
Triangular .	Rectangular .	$0{\cdot}68 \times 10^{-6}$	$-0{\cdot}82 \times 10^{-3}$

Mutual Induction.—This is generally investigated from the two well-known simultaneous equations. Let V be the instantaneous volts across the load, e the instantaneous volts across load plus series coil. Let R, L and M be the resistance self-induction of the series coil and M mutual induction, then for a balance we must have

$$e = \mathrm{V} + i_1\mathrm{R} + \mathrm{L}_1\frac{di_1}{dt} + \mathrm{M}\frac{di_2}{dt},$$

where i_1 is the instantaneous current in the series coil, and i_2 that in the shunt coil. Similarly for the shunt coil, we have

$$e = \mathrm{S}i_2 + \mathrm{L}_2\frac{di_2}{dt} + \mathrm{M}\frac{di_1}{dt}.$$

Multiply both by i_1,

$$ei_1 = \mathrm{V}i_1 + i^2\mathrm{R} + \mathrm{L}_1 i_1\frac{di_1}{dt} + \mathrm{M}i_1\frac{di_2}{dt},$$

and
$$ei_1 = i_1 i_2\mathrm{S} + \mathrm{L}_2 i_1\frac{di_2}{dt} + \mathrm{M}i_1\frac{di_2}{dt}.$$

Let
$$\mathrm{A} = \text{R.M.S. value of } ei,$$
$$w = \text{R.M.S. value of } \mathrm{V}i_1,$$
$$i^2\mathrm{R} = 0,$$

and since
$$\frac{\mathrm{L}_1}{\mathrm{T}}\int_0^{\mathrm{T}} i_1\frac{di_1}{dt} = 0 \text{ approximately,}$$

and
$$\frac{\mathrm{L}_2}{\mathrm{T}}\int_0^{\mathrm{T}} i_1\frac{di_2}{dt}$$

is supposed not negligible, we have

$$\mathrm{A} = w + \frac{\mathrm{M}}{\mathrm{T}}\int_0^{\mathrm{T}} i_1\frac{di_2}{dt}dt$$

and
$$\mathrm{A} = \mathrm{I}_1\mathrm{I}_2\mathrm{S}\cos\phi + \frac{\mathrm{L}_2}{\mathrm{T}}\int_0^{\mathrm{T}} i_1\frac{di_2}{dt}dt,$$

$$\therefore \quad \frac{\mathrm{T}}{\mathrm{M}}(\mathrm{A} - w) = \left(\frac{\mathrm{A} - \mathrm{I}_1\mathrm{I}_2\mathrm{S}\cos\phi}{\mathrm{L}_2}\right)\mathrm{T}.$$

$$w = \mathrm{A}\left(\frac{\mathrm{L}_2 - \mathrm{M}}{\mathrm{L}_2}\right) + \frac{\mathrm{I}_1\mathrm{I}_2\mathrm{SM}\cos\phi}{\mathrm{L}_2}.$$

But $SI_1I_2 \cos\phi = k\delta$, the reading of the instrument. $T =$ the periodic time.

A is the average power of the whole circuit. By making $M = L_2$, $w = k\delta$.

This calculation assumes that M remains constant, which is not the case in a moving coil instrument. A discussion of the coefficients of mutual induction for coaxial coils will be found in Gray's *Absolute Measurements*, vol. ii. part ii., the whole subject being very complex.

Mutual Induction Error.—Since the mutual energy T of two coils can be expressed by the series

$$T = \pi^2 I_1 I_2 \frac{a_1{}^2 a_2{}^2}{r^3} \{1 . 2 . \cos\phi + 2 . 3 . \frac{x}{r^3} \zeta (\cos^2\phi - \tfrac{1}{2}\sin^2\phi) + \text{etc.}\},$$

which is obtained by means of spherical harmonics, and since $M = \frac{T}{I_1 I_2}$ we might assume that for a pair of coils in a wattmeter

$$M = k \cos\theta,$$

where k is a constant and θ is the angle between the axis of the coils. For certain dimensions of the coils, *vide* Gray's *Absolute Measurements*, vol. ii. part i. p. 276; the calculation of M from the first term of a series given there gives an extremely accurate result, and it is pointed out this should be taken into consideration in dynamometer construction.

We see then that if we introduce this into the equation on p. 190, we have

$$w = A\left\{\frac{L_2 - k\cos\theta}{L_2}\right\} + \frac{I_1 I_2 SM \cos\phi}{L_2}.$$

But the second term on the right hand side is the watt-meter reading, or $K\theta$, so that

$$K\theta = w - A(1 - \frac{k}{L_2}\cos\theta).$$

We see, therefore, that we cannot make the error zero for all positions of moving coil instruments. In zero instruments $\theta = \frac{\pi}{2}$, and this error disappears to a first approximation if $\frac{k}{L_2} = 1$.

The error due to the earth's magnetism is noticed in the zero dynamometer pattern on passing a current through the fine wire coil if it is not lying in the magnetic meridian. The couple due to the earth's magnetic force vanishes when balanced by torsional zero, if the instrument is correctly placed. In moving coil instruments of the dial pattern, the effect of the vertical intensity of the earth's magnetism is too small to cause much error compared with the other forces acting.

Instruments like Kelvin's engine-room wattmeter have two moving and two fixed coils, so that this error is entirely eliminated.

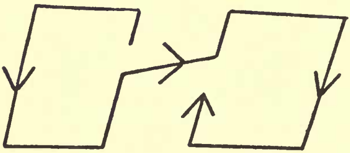


FIG. 66.—Moving coil of astatic wattmeter.

Eddy Currents.—The effect of metal acted upon by the fields is to increase the equivalent resistance, and to diminish the equivalent inductance of the wattmeter circuit. Metal, however, is rarely in the vicinity of the fields.

The electrostatic attraction is generally quite negligible.

A more serious trouble, especially with wattmeters of this variety, is the fact that the law of attraction for heavy conductors in close proximity to a coil is not known, since the distribution of current in the conductor is uncertain. This seems by far the most important matter to be considered.

A well-designed wattmeter should not have an error exceeding 0·001 in the power factor, and this is a possible figure within certain limits.

Inaccurate Wattmeter

When the power factor is low, a wattmeter may be inaccurate, but, as pointed out by R. Beattie (*Electrician*, March 14, 1902, "Test Coils for Alternating Current Wattmeters"), if the actual reading D_a and true reading D_t are connected by the formula

$$D_a = D_t\left(1 + \frac{Lw}{R}\tan\theta\right),$$

where L is the coefficient of self-induction and R the resistance of the fine wire coil, D_a is a linear function of L. Consequently, by varying L, a series of points lying in a straight line are obtained. If, however, this line is produced backwards till it cuts the ordinate, the true reading is obtained. By putting a test coil of equal inductance to the wattmeter pressure coil in circuit, the reading will be altered by an amount +ve or −ve just equal to the error.

The reactance of the fine wire coil must be small,

otherwise the linear relation referred to above will not hold. In the case of a non-sinusoidal E.M.F. wave the method would still give correct results, since the relation

$$D_a = D_t \left\{ 1 + \frac{Lw}{R} \frac{E_1 I_1 \sin\theta_1 + 2E_2 I_2 \sin\theta_2 +, \text{ etc.}}{E_1 I \cos\theta_1 + E_2 I_2 \cos\theta_2 +, \text{ etc.}} \right\}$$

is still linear.

If the circuit in which the power developed consists of a self-inductance in series with a resistance, the percentage error is proportional to

$$\frac{E_1^2 + E_2^2 + E_3^2 +, \text{ etc.}}{E_1^2 + \frac{E_2^2}{4} + \frac{E_3^2}{9} +, \text{ etc.}}$$

A self-inductance in parallel with a resistance makes the error independent of wave form and frequency.

A condenser in parallel with a high resistance increases the error, the percentage varying as

$$\frac{E_1^2 + 4E_2^2 + 9E_3^2 +, \text{ etc.}}{E_1^2 + E_2^2 + E_3^2 +, \text{ etc.}},$$

but with a condenser in series with resistance, the error is independent of wave form and frequency.

The Induction Ammeter

These instruments are generally arranged as in Figs. 67, 68, 69. As is explained in dealing with the induction meter (*vide* Chapter, Supply Meters), the flux due to the

current on the magnet coils induces an eddy current in

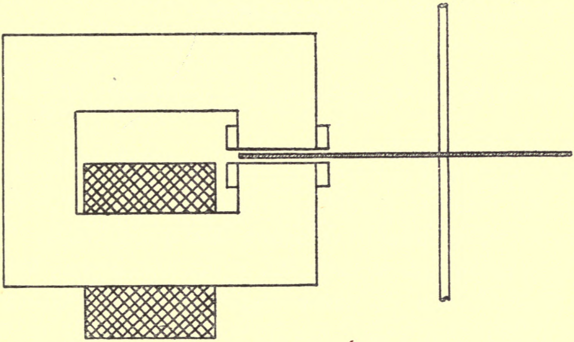


FIG. 67.—Induction ammeter.

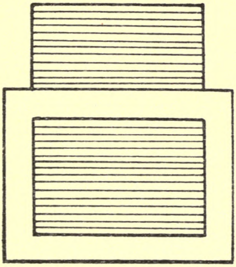


FIG. 68.—Pole face showing shading coil.

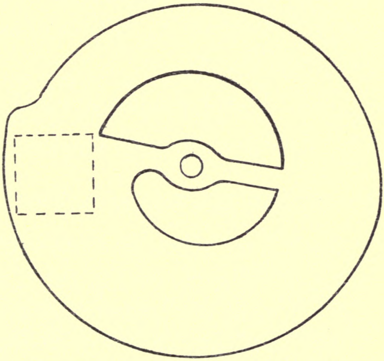


FIG. 69.—Disc in undeflected position. Direction of motion clockwise; pole face dotted.

the disc 90° out of phase with the flux. If this is so, then

$$\Sigma(\text{flux} \times \text{eddies in disc}) = 0$$

for a complete period.

The currents induced, however, in the shading ring

and disc will be nearly 180° out of phase with one another, and will repel one another.

On account of the large air gap the magnetising current is large, and will vary approximately as $\frac{1}{n}$ where n is the periodicity.

By putting a suitable resistance in parallel with the coil, the vector sum of the coil current + current in shunt is kept approximately constant over a wide range.

It appears that the current in the disc lags ϕ_1 behind the E.M.F., and its phase displacement in relation to flux is $(90 + \phi_1)$ (*vide* "Eddies in Moving Coil," p. 78).

The current in the shading ring lags $(90 + \phi_2)$. By current is meant the eddy or sum of the filamentary current streams in the two cases. Consequently the torque is due to

$$\text{Flux} \times \{\text{eddies in disc}\} + \text{eddies in disc} \times \{\text{eddies in shading ring}\}.$$

As periodicity increases the flux $\propto \frac{1}{\text{periodicity}}$ if the voltage is constant, whereas neglecting magnetic leakage the voltage generated in the disc is constant if we consider it from the transformer standpoint.

So that for constant voltage on the magnetising coil

$$\text{Flux} \propto \frac{1}{n},$$

$$\text{Eddy in disc} \propto \frac{\text{constant}}{\text{impedance}},$$

and these become more nearly in phase as the periodicity increases.

Actually, as shown below,

$$e = an + b,$$

where a and b are constants.

$$a = 0{\cdot}005, \ b = 0{\cdot}453$$

for a wide range in the instrument tested.

Again, if volts on magnetising coil are constant, the eddy currents in disc and shading ring

$$\propto \frac{\text{E.M.F.}}{\text{impedance}},$$

in each case ; ϕ_1 and ϕ_2 and the impedance of disc and ring increase. The torque will therefore vary with

$$\cos (\phi_1 - \phi_2).$$

Since the shading ring is much more massive than the disc, ϕ_2 cannot equal ϕ_1, so that apparently we require increased voltage drop at increased frequency, which agrees with tests.

Temperature Error.—On the magnetising coil this will be small, since its resistance is only about ·023 ohms. If the resistance of disc and shading ring increases, the currents diminish, but ϕ_1 and ϕ_2 will also decrease. To compensate for this the resistance shunting the magnet winding may be of copper of such gauge that its resistance increases at the same rate as that of the disc or other portions. Consequently, a rise of temperature would allow more current to pass through the magnetising coil and compensate for any increase of disc and ring resistance.

The following tests were made on an ammeter of this type :

First the volt drop across the instrument alone (without shunt) was measured and found to be, for a given scale reading (5 amperes),

$$v = 0{\cdot}005n + 0{\cdot}453$$

over the range. A test was then performed as indicated in Fig. 70, and the table gives the observed

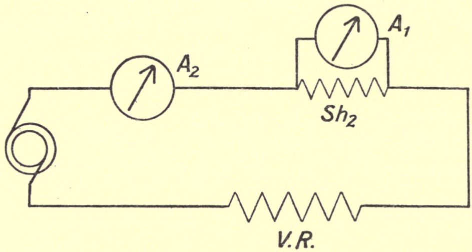


FIG. 70.—Test on induction ammeter.

A_1, Coil of instrument.
A_2, Ammeter reading total current.
Sh_2, Resistance in parallel with instrument coil.
V.R., Variable resistance.

results together with those calculated from the vector diagrams.

The following tests were made :

The instrument reading was kept constant at 5 amperes and the true current noted, the periodicity being varied, and the results below were obtained.

[TABLE

READING OF INSTRUMENT (Unshunted)

5 Amperes.		8 Amperes.	
True Current.	Periodicity.	True Current.	Periodicity.
3·88	30	4·80	45
3·40	40	4·48	50
3·18	45	4·03	60
3·00	50	3·80	70
2.82	60	3·62	80
2·70	70	3·50	90
2·59	80	...	...
2·49	90	...	...

READINGS

Speed. 8-Pole Machine.	Total Current observed.	Total Current calculated.	Resistance in Parallel.
715	5·05	...	·2 ohm
890	5·00	...	...
1015	5·00	...	...
1080	5·05	...	...
1140	5·10	...	...
1280	5·20	...	...
1470	5·6	...	...
1370	5·35	...	...
620	5·20	...	...
640	4·3	...	·3 ohm
855	4·2	...	...
1010	4·0	...	...
1080	4·00	...	...
1140	4·00	...	...
1225	4·00	...	...
1400	4·10	...	...
1470	4·10	...	...

READINGS (*continued*)

Speed. 8-Pole Machine.	Total Current observed.	Total Current calculated.	Resistance in Parallel.
610	4·1	4·08	0·4 ohm
740	3·8	3·79	...
890	3·6	3·62	...
1010	3·5	3·53	...
1200	3·45	3·45	...
1340	3·5	3·49	...
1160	3·45	3·46	...
1020	3·5	3·52	...
810	3·7	3·70	...

Vector diagrams were drawn, from which the phase displacements between current in the instrument and

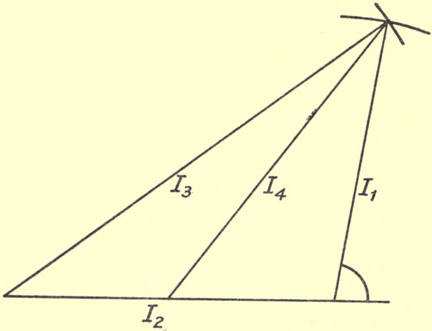


FIG. 71.—Vector diagram for 50 periods.

I_1, Current through coil, 3·00 amperes.
I_2, Current through shunt of ·2 ohms, 3·51 amperes.
I_3, Total current, 5·05 amperes.
I_4, Total current with shunt of ·4 ohms.
From diagram, 3·77 amperes ; observed, 3·78 amperes.
ϕ, Phase displacement between voltage across and current in coil $=78{\cdot}5°$.

volts between its terminals were determined for the first set of readings, and from these the total current

for parallel resistances of 0·3 and 0·4 ohms were calculated.

These results are given in the following table :

VALUES READ OFF VECTOR DIAGRAM AND CURVE

Periodicity.	Total Current.		
	·2 ohm. Off Curve.	·3 ohm. From Diagrams.	·4 ohm. From Diagrams.
40	5·2	4·40	4·10
45	5·12	4·25	3·90
50	5·05	4·16	3·77
60	5·00 min.	4·05	3·62
70	5·04	4·00 min.	3·52
80	5·11	4·00	3·45 min.
90	5·33	4·05	3·50
100	5·65	4·25	3·60

The following table exhibits the change in phase angle between current and volts in the magnetising coil :

[TABLE

PHASE ANGLE IN COIL

Periodicity.	Angle.
40	76
45	77
50	78·5
60	82
70	84·5
80	87
90	86
100	82·5

It will be noticed the phase angle increases up to a maximum of 87° and then decreases as the frequency increases.

It will be seen also that as the periodicity increases, if the current read on the instrument be kept constant at 5 amperes, the true current decreases with periodicity, the phase angle first increases and then diminishes. The total watts absorbed at 40 periods is of the order of about 0·5 watts.

After a two hours' test at 5 amperes the heating was hardly perceptible.

CHAPTER VIII

SUPPLY METERS: GENERAL

THE meters dealt with in this chapter are of the types used to measure electrical energy. As a rule they are of two classes, viz. Coulomb Meters and Watt Hour Meters.

In this country, and more recently in France, coulomb meters have been largely used, whereas in America and the rest of Europe watt hour meters are more common.

Since coulomb meters measure It they take no account of any pressure variations except so far as this may increase the current to a consumer.

The power supplied to a consumer is EI, and the energy EIt. If the resistance of the installation supplied be regarded as constant for all currents, the energy supplied is

$$E^2/R \times t.$$

Suppose now the voltage varied by ΔE. The energy measured by the watt hour meter is

$$\frac{(E \pm \Delta E)^2}{R} t,$$

and by the coulomb meter

$$\frac{E(E \pm \Delta E)}{R} t.$$

Neglecting squares of small quantities, we have for the difference of energy registered

$$\frac{\pm 2\Delta E \,.\, E \mp \Delta E \,.\, E}{R} t,$$

or

$$\pm \frac{\Delta E \,.\, E}{R} t.$$

This would correspond to the difference in readings of Board of Trade Units of the two meters.

If this quantity is positive, the Supply Company is losing by this amount if the consumer uses a coulomb meter, and when negative the consumer is being charged too much, to this extent. In an installation taking 200 kilowatt hours per annum a 1 per cent error means an excess or deficit of 2 kilowatt hours per annum.

To a considerable extent this error in supplying is met by the fact that the error is probably as often positive as negative, so that on the whole they balance.

We see, however, that there can be no question that the watt hour method is the more correct.

There is however another aspect of the systems of metering. If with a watt hour meter the energy is given by

$$W = k_1 \frac{E^2}{R} t,$$

any error in the meter in measuring E will cause an error in W, thus

$$\frac{dW}{W} = \pm \frac{2dE}{E},$$

whereas if a coulomb meter is used the energy is given by

$$W = k_2 \frac{E}{R} t,$$

and the error

$$\frac{dW}{W} = \pm \frac{dE}{E}.$$

So that if R is constant and the current in the coulomb meter is measured with the same degree of accuracy as the currents on the watt hour meter there is an advantage in favour of the coulomb meter in the ratio of two to one.

The two types of meters may be divided into the following classes :

I. *Coulomb meters*—1. Electrolytic meters.
2. Mercury meters.
3. Motor meters.
4. Pendulum meters.

II. *Watt hour meters*—1. Motor meters.
2. Pendulum meters.
3. Mercury meters.

The principles involved in the design of electric meters are generally of an exceedingly simple character ; the difficulty consists in so arranging the working part that it gives strictly straight line laws.

Electrolytic Meters.—These depend for their action on Faraday's Laws of Electrolysis, or

$$w = eQ,$$

where w is the weight of substance or substances decomposed, e the electro-chemical equivalent, Q the quantity of electricity.

If $Q = It$, then the meter is a coulomb meter.

As a rule, such meters have a back E.M.F., e, say, which makes $I = \frac{E - e}{r}$, so that the meter registers, where r is the total resistance of meter and installation,

$$\frac{E - e}{r} t = It.$$

The meter is correct, therefore, as a coulomb meter.

The energy supplied to the installation is $(E - e)It$, and the total energy supplied by the supply mains is

$$\frac{E^2}{r} t, \text{ or } EIt.$$

By subtraction we see that the energy wasted in the meter is

$$eIt,$$

and since the meter is calibrated for E the consumer has to pay for the above quantity.

Mercury Meters.—In these the rotation of mercury is made proportional to the current, and this rotation by driving a vane works a train of wheels which integrates It.

Pendulum Meters.—These may be watt hour meters or coulomb meters. In Lord Kelvin's original meter a solenoid was used attracting a magnetically " saturated " plunger into it. The plunger was made of a thin rod of soft iron. Periodically the plunger was pulled up to the zero position by means of a spring, and the distance moved over, which was strictly proportional to the current or energy, was metered by a train of wheels.

The periodic times of lifting were fixed by means of a pendulum.

Motor Meters.—In these meters the number of revolutions must be made proportional either to coulombs or watt hours.

In such commutator meters as Elihu Thomson's, the motor contains no iron and, consequently, works satisfactorily as an energy meter on both continuous or alternating circuits.

Meters are necessarily integrators,

Coulomb meters giving $Q = \int C dt$

and Watt hour meters giving $\int eidt$.

In all motor meters it may as well be noted the condition for steady running at constant speed is derived from the equation

$$\frac{d^2\theta}{dt^2} = \frac{\text{moment of accelerating forces} - \text{moment of retarding forces}}{\text{moment of inertia}},$$

where $\frac{d^2\theta}{dt^2} = 0$ when the speed is constant. Hence the condition that

Moment of accelerating forces = Moment of retarding forces,

as a rule gives the relation between watts and speed, or coulombs and speed, as the case may be.

In the following pages many applications of this equation in some form will be found.

Errors in Meters : *Electrolytic Meters.*—As a rule these meters, owing to the small amount of chemical decomposition permissible, are difficult to read accurately. When shunted, since the electrolyte has a negative temperature coefficient of resistance, it is impossible to compensate it accurately. The electrolyte may evapor-

ate or creep up the electrode or their connections. An objection is the loss of record on resetting the meter, the difficulty of resetting and the loss of energy in the meter itself, owing to the back E.M.F. of polarisation when current is passing through the meter.

Mercury Meters.—Those meters which use mercury suffer from amalgamation and a gradual disintegration of the mercury (*vide* Ratcliffe and Moore, *Journal of I.E.E.* vol. xlvii. p. 6); at the same time meter manufacturers are prepared to guarantee these meters at a small annual cost.

Mercury meters for 50 to 100 amperes also of the series type suffer from what appears to be solution of the copper in mercury, the copper being deposited at the cathode. Temperature also affects these meters, and this may be reduced by shunting.

Commutator Type Ampere Hour Meters.—In these meters the resistance of the "contact" on the commutator may vary, especially at low loads (*vide Elec. Rev.* 16.vi.11, "Commutator Resistance and Energy Losses," W. H. F. Murdoch). In these meters a permanent magnetic field is used, and this may vary with time, diminishing the armature torque, and making the meter read too low. Frequently on account of space considerations the magnets are made too short for permanency, or are affected by short circuits.

Fluctuating loads may cause errors in some classes of meters.

Pendulum meters of the Aron type are peculiarly free from errors; a load synchronising with the periodic time introducing resonance trouble is practically impossible under ordinary working conditions.

Other points to be kept in view in meter design are

troubles due to wear and tear, possibly due to too high a speed, delicate construction, difficulty, and cost of making repairs, troubles with bearings.

Alternating Meters.—These again should be independent of wave form and frequency. The Aron clock meter is practically so.

The other types of alternating meters are either of the dynamometer type with commutator like Elihu Thompson's meter or meters of the induction type. The former are probably most suitable for continuous heavy work, but the latter are much smaller, lighter and cheaper. The induction type suffers perhaps from the fact that the driving torque is small, and is not independent of wave form. However, it can be accurately adjusted for frequency and wave form. It also is independent of temperature, since eddy currents of the brake are generated on the same disc as the driving currents and so cancel out of the equation.

Stray Fields also cause trouble to some meters, and here the astatic principle is sometimes used, as in the Aron meter both pendulums are equally affected.

These troubles will be discussed in due course in considering the designs of the various types.

Electrolytic Meters: General

These are all naturally coulomb meters working in accordance with Faraday's laws, as already stated (p. 205). The current is supposed to flow through the electrolyte and follow the law

$$C = \frac{E - e}{r},$$

where E is the E.M.F. applied to the cell, e the back E.M.F. of polarisation, and r the resistance of the cell.

Generally speaking, polarisation is associated with the fact that often on stopping a current flowing through an electrolytic cell it is found to possess an E.M.F. of its own always in the reverse direction to the one applied. In electric meters this E.M.F. is assumed to be either small or constant in value.

Le Blanc also found that as the applied E.M.F. is raised in value, the back E.M.F. also increases, and if on further increase of E.M.F. no further rise takes place in the B.E.M.F., then the expression

$$\frac{E - e}{r}$$

may be used to calculate the current.

Le Blanc (*vide Electro-Chemie*) found that for certain salts there is no apparent limiting value of this polarisation E.M.F. For the method of measuring the back E.M.F. his treatise must be consulted.

It appears that for solutions of metals the maximum decomposition voltages not only vary for each substance, but have no common value.

For zinc sulphate it is 2·35 volts,
For silver nitrate it is 0·70 volts,
For cobalt chloride it is 1·78 volts,

and so on.

The decomposition voltages for copper sulphate is $-0{\cdot}515$ volts and zinc sulphate $+0{\cdot}524$ volts.

Acids and alkalies appear to give the maximum decomposition voltage independently of concentration.

Sodium hydrate is 1·69 volts.
Potassium hydrate is 1·67 volts.
Sulphuric acid is 1·67 volts.
Nitric acid is 1·69 volts.

For the solution of metals examined by Le Blanc, he found that the maximum decomposition voltage at the cathode (on which metal is being deposited) equals the P.D. between a plate of the metal and the solution in contact with it.

Again in cases where platinum electrodes are used, the thermodynamic considerations applied to Grove's gas battery, for example, for oxygen and hydrogen, will apply also to polarisation (see Whetham, *Theory of Solution*, p. 305). In such a case

$$\mathrm{E} - e = \frac{\mathrm{RT}}{qy} \log_\epsilon \frac{p_1}{p_2},$$

where R is the gas constant, T the absolute temperature, q the charge of electricity passing per equivalent gramme cm. liberated, y the valency of the ions, and p_1 and p_2 the pressures in the two cases. For low values of p_2 we have low values of e, the reverse electromotive force.

In primary batteries, various means are adopted to get rid of polarisation—Smee's battery with platinised silver plates, Daniell's two fluid, and Leclanché are examples. Again, in measuring resistances containing a polarisation E.M.F., the rapidly alternating current and telephone is often used. In meters, however, its elimination seems at present impossible.

There are several meters of an interesting type which may be briefly referred to.

In the Mordey Fricker meters of the prepayment type,

inserting a coin enables a certain length of copper strip to be inserted in liquid. This is gradually eaten away by the current. The copper strip is unwound from a roll so that it is eaten away from the bottom upwards. The strips are kept from touching by glass separators.

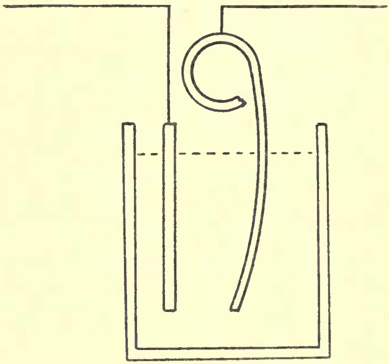


FIG. 72.—Mordey Fricker Meter.

The principle of loss of weight of electrode being used as an indication was in use in 1882, when a meter registered the loss on a dial graduated in ampere hours.

One such type of meter is so arranged that water is decomposed electrolytically. This lightens the meter, which is suspended by springs; its upward movement causes it to register the motion by means of a rack and pinion working an index over a dial fixed outside the meter proper. The working will be understood from Fig. 73.

The accuracy of the meter could be checked at any time by placing a weight on the meter, and so calibrating it gravitationally; the initial calibration by altering the strength of the springs, by varying the divisions on the dial, or by varying the number of teeth in the pinion. It contained about $4\frac{1}{2}$ lbs. of water, and when $1\frac{1}{2}$ lbs. were decomposed, it was refilled. This corresponds to a registration of 400 or 500 units at 100 volts. Other

meters have been contrived so that alteration of the density of the solution by electrolysis (produced by

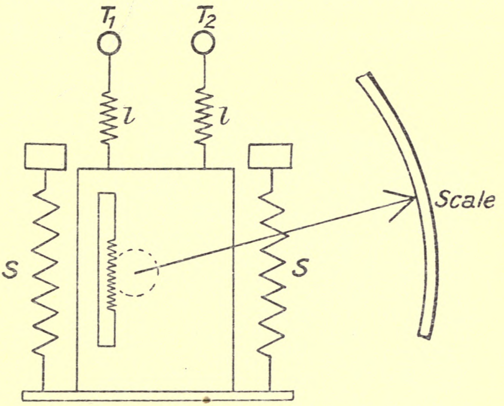


FIG. 73.

S, S, Springs. l, l, Flexible leads. T_1, T_2, Terminals.

electrolysis of water) is registered by means of a submerged float on a dial.

Mr. S. H. Holden (*Jour. I.E.E.* vol. xxxvi. p. 393) has devised a meter in which A and C are electrodes of platinum immersed in dilute sulphuric acid contained in the vessel S. Above the electrodes is a sealed space containing hydrogen H. If, then, oxygen is evolved at the +ve pole, and hydrogen at the −ve, the anode absorbs H from the store to neutralise the oxygen, the hydrogen liberated passing

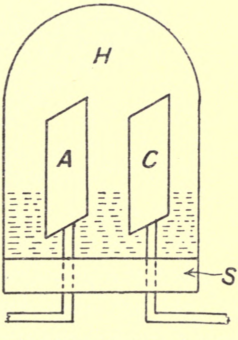


FIG. 74.—Holden Meter.

into a measuring tube. This meter was used shunted, and tilting the meter refilled the measuring tube with water, and it could then be used again. Its action will be sufficiently understood from Fig. 75. It will be

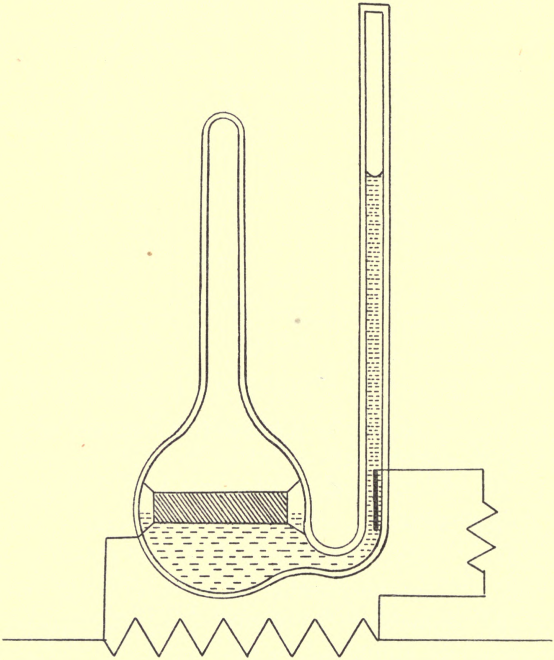


Fig. 75.—Holden Meter.

noticed the principle involved of continuous absorption of gas at one electrode and evolution at another was a most interesting application.

Prepayment Devices.—An interesting one intended as a prepayment meter for use in hotels and such places

with electric radiators in rooms has been placed on the market by Drake & Gorham, Ltd. It consists of a time switch which can be set to operate at any predetermined time. When it is desired to use the radiator a coin is

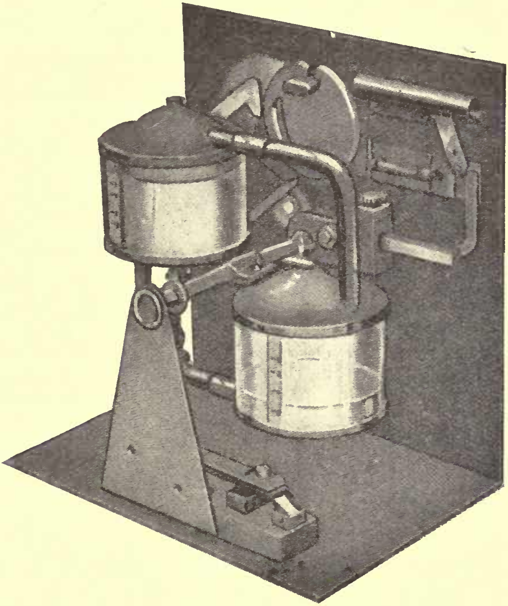


FIG. 76.—Prepayment device.

inserted in a slot, and the radiator can be used for the time arranged. At the end of the time the current is automatically switched off by gravity. The time is settled by the rate of flow of liquid from a higher to a lower chamber. The apparatus can be set for any desired period between one and ten hours.

Wright Meter

This meter, which is of the "shunted electrolytic" type, uses potassium mercuric iodide and mercury and iridium electrodes.

The connections will be understood from Figs. 77 and 78.

When a current passes through the electrolyte, metallic mercury is deposited on the cathode and fills into the measuring tube in minute globules. When the measuring tube is full, this syphons into another vessel. In the course of

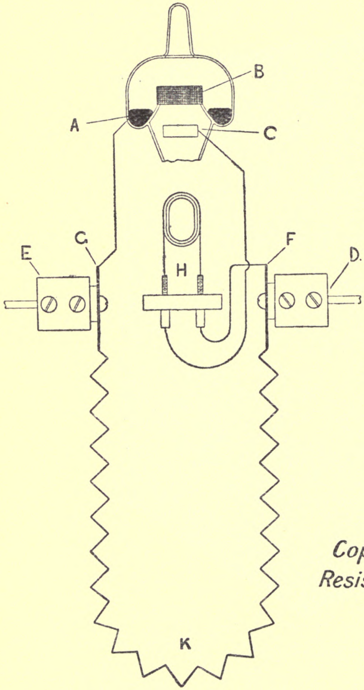


Fig. 77.—Wright Meter.

A, Mercury anode. D, E, Terminals.
B, Platinum gauze fence. F, G, K, Shunt.
C, Iridium cathode. H, Copper resistance.

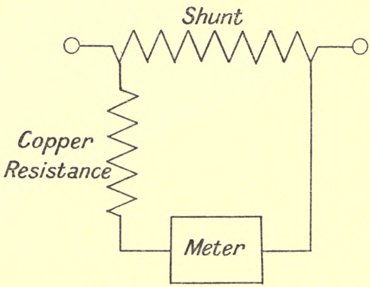


Fig. 78.—Connections of Wright Meter.

time this tube fills up, and to reset the meter the mercury is tilted back into the reservoir again. This

will be understood from Fig. 79. In such a case as this, the chief points are back E.M.F., and constancy of the ratio

$$\frac{\text{Current through cell}}{\text{Current in main circuit}}.$$

Now the current which passes through the electrolyte is

$$i = \frac{E - e}{r} \text{ or } E = e + ir.$$

But $E = (I - i)R$, the drop of volts across the shunt. Hence

$$(I - i)R = e + ir,$$

$$\therefore \quad \frac{IR - e}{r + R} = i,$$

and the ratio

$$\frac{i}{I} = \frac{IR - e}{I(r + R)},$$

or

$$\frac{i}{I} = \frac{R - \frac{e}{I}}{(r + R)},$$

or

$$= \frac{R}{R + r} - \frac{e}{I(r + R)}.$$

This has to be constant. It is claimed that the back E.M.F. is less than $\frac{1}{10000}$ volt owing to gravitational stirring.

Assuming, however, that it is so, the ratio of currents is the ratio $\frac{R}{r + R}$.

And since r is an electrolyte, its coefficient of resist-

ance variation with temperature is negative, whereas that of R is positive.

In this meter a fine wire resistance of copper in oil is placed in series with the electrolytic cell, so that a drop in resistance of the electrolyte is compensated by an increase in the copper resistance. If a_1 be the tempera-

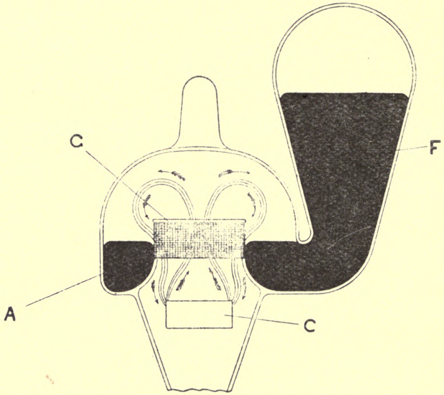


FIG. 79.—Wright Meter showing convection currents.

A, Mercury anode. F, Anode feeder.
C, Iridium cathode. G, Platinum gauze fence.

ture coefficient for copper, β that for the solution, then for any temperature θ we have

$$\frac{r_0(1+a_1\theta)+r_1(1-\beta\theta)}{R(1+a_2\theta)},$$

where r_0 is resistance of series coil, r_1 of the electrolyte, and R the shunt. Since a_2 is a negligibly small quantity, we have the necessary condition for complete compensation,

$$r_0 a_1 = r_1 \beta.$$

This meter is now made to measure as follows for the different sizes :

$2\frac{1}{2}$ ampere
5 ,,
10 ,,

The Wright meter is a decidedly interesting meter, and one of the very best electrolytic meters.

In order to keep the level of the mercury constant as the decomposition proceeds, a bird-cage fountain arrangement is adopted. If p_1 is the negative pressure inside the fountain, $\rho \mathrm{H} g$ the pressure due to the head of mercury, then

$$\rho \mathrm{H} g - p_1 = \text{constant}.$$

Consequently, if H, the height of the mercury column, is altered, p_1 is diminished, so that the difference is constant. In other words

$$\frac{d\mathrm{H}}{dp_1} = \text{constant},$$

this constant head being the pressure due to the solution on surface of mercury anode.

Electrolytic Resistance

Owing to the presence of polarisation E.M.F.'s, the measurement of the resistance of electrolytes is generally more complex than that of ordinary resistance.

It is shown in treatises (such as Whetham's *Theory of Solution*) that if u and v are the ionic velocities of the

ions in a simple dilute solution such as HCl, the volt drop per centimetre $\frac{dV}{dx}$ is

$$\frac{mq}{k}(u+v)=\frac{dV}{dx},$$

$k\frac{dV}{dx}$ represents the current, so that k measures the conductivity or the reciprocal of the specific resistance, q is the charge per ion per gramme equivalent, $m(u+v)$ are the numbers of gramme equivalents liberated.

For one volt per centimetre, n the concentration of the solution,

$$\frac{k}{n}=\frac{(u_1+v_1)\,.\,100}{1{\cdot}037},$$

the quantity q being taken as 96,440 coulombs. Hence, as is usual, by measuring the conductivity u_1+v_1 is determined, and if Hittorf's migration ratio $\frac{u}{v}$ is known accurately, the ionic velocities can be calculated. The velocities of migration can also be observed by measuring the movement of the line of demarcation between the ions as in Lodge's experiment with a coloured liquid, or as in Dr. B. Steele's method, where the change of refractive index gives a sharp boundary in a transparent liquid.

To measure the conductivity then, either a Kohlrausch Bridge method with induction coil and telephone is used, or a galvanometer with a reversing switch and rectifier, so that the deflection of the moving coil D'Arsonval galvanometer is always in the same direction. This is in many ways preferable to the telephone, because when a

telephone and induction coil are used there is never perfect silence, merely a reduction to minimum of the sound. To enable one to sharply adjust the balance the vibrator should have a period of about 1000, emitting a sharp " gnat-like " sound rather than a lower frequency note.

When using the preferable arrangement with reflecting galvanometer, polarisation is considered eliminated when, on increasing the speed of the reversing switch, no change is produced with galvanometer deflection. The moving coil galvanometer must also have sufficient movement of inertia not to be sensibly effected by the current pulsations.

A method adapted for ordinary testing is that due to Stroud and Henderson. Here two tubes of the solution to be tested with similar electrodes are placed as arms in a Wheatstone bridge. A resistance is placed in series with the shorter tube, and, when zero is obtained, the value of r gives the resistance due to the difference in length of the tubes, or

$$r = \frac{l_1 - l_2}{\text{area}} \times \rho,$$

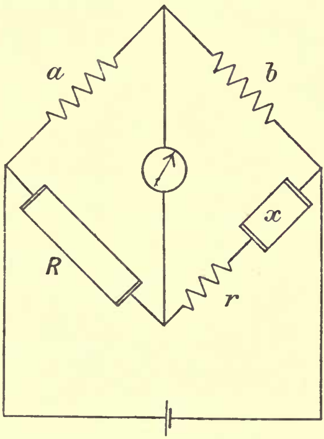


Fig. 80.—Stroud and Henderson's method.

so that the specific resistance ρ can be calculated. The polarisation is assumed the same in both cases, and since it acts in opposite

directions cancels out. As a rule, some trouble is experienced with polarisation, since during the process of balancing the currents are not equal in the two tubes, and sometimes after balance is obtained the galvanometer reading, instead of remaining steadily at zero, begins to creep gradually over the scale. Electrodes of platinum black are generally used in all cases and have to be carefully prepared.

For many purposes, rough measurements of resistance can be made, as with iron electrodes in $NaOH$, say, by first measuring e with a voltmeter, and the current passing, and then taking a series of readings rapidly with increasing currents. In this case on plotting, a straight line is generally obtained from which, by producing it to cut the vertical axis, e may be determined,

since $$E = e + Ir.$$

Hence $$r = \frac{E - e}{I},$$

or the tangent of the angle made by the line with the horizontal through e gives r. Knowing the dimensions of the vessel, which is generally of rectangular section, to constrain the stream lines,

$$r = \frac{\rho l}{A},$$

and ρ can be found.

It must not be forgotten in dealing with liquid resistances that great attention must be paid to the shape and position of the electrodes. A discussion of several cases is given in Gray and Matthew's *Bessel's Functions*.

Generally speaking, the effects of polarisation are very complicated, but if V is the voltage at the terminals of an electrolytic cell, i the current, P a polarisation constant depending on nature of electrolyte, area of plates and kind of electrodes, then

$$V = Ri + P\int i\,dt.$$

If V is assumed a sine function (which it is not generally), then

$$Ri + P\int i\,dt = V \sin pt,$$

differentiating

$$R\frac{di}{dt} + Pi = pV \cos pt,$$

so that the solution is obviously

$$i = \frac{V}{R\left\{1 + \frac{P^2}{p^2R^2}\right\}^{\frac{1}{2}}} \sin(pt + \phi),$$

where

$$\phi = \tan^{-1}P/pR, \text{ and } p = 2\pi n.$$

We see that to diminish the effects of polarisation, P should be small and p as large as possible, and then

$$i = \frac{V}{R} \sin pt \text{ approximately.}$$

Electrolytic Meter: Long Schnattner

This is a prepayment meter in which weight is lost by a copper plate attached to a lever. When this takes place the lever rises, and after a time a coin is dropped

into a receptacle attached to the lever, bringing it into equilibrium again. If the coin is not dropped in at the proper time a dimming resistance is inserted in the circuit, thereby indicating by the diminished candle power that more money is required. The liquid used is copper sulphate 1·08 density with 1 per cent of sulphuric acid added. The copper lost from the plate above referred to is deposited in the containing vessel.

The theory is very simple.

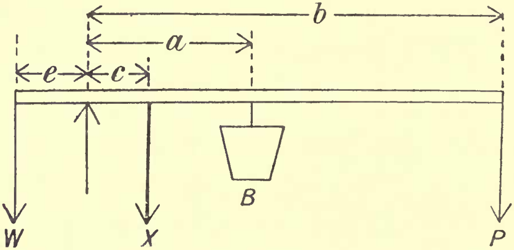


Fig. 81.—Long Schnattner Meter.

Let W be the balance weight, e its arm to the fulcrum, B the money-box and a its arm, P the weight of plate and b its arm. The centre of gravity of beam is at a distance c from fulcrum, and we shall call the weight X.

For equilibrium the initial condition with no money in the box is

$$\mathrm{W}e = a\mathrm{B} + \mathrm{X}c + \mathrm{P}b\left(1 - \frac{d_0}{d}\right),$$

since $\frac{\mathrm{P}d_0}{d}$ is the weight of copper sulphate displaced by the plate P, d_0 the density of copper sulphate, d the density of copper.

Now let there be n coins of weight w each, and the

weight of plate lost $I\epsilon t$, where I is current, ϵ the electro-chemical equivalent, t the time, then we have

$$We = aB + Xc + Pb\left(1 - \frac{d_0}{d}\right)$$

$$+ wna - bI\epsilon t\left(1 - \frac{d_0}{d}\right),$$

$$\therefore \quad wna = bI\epsilon t\left(1 - \frac{d_0}{d}\right).$$

We see that wn, the weight of coins in the box, is proportional to It, since a, b, $\frac{d_0}{d}$ and ϵ are constants.

$$\therefore \quad £ . S . D \propto Q,$$

where Q is the quantity of electricity.

Temperature Error.—Since a and b both expand at the same rate, being of the same material, we have

$$nwa(1 + a\theta) = I\epsilon tb(1 + a\theta)\left(1 - \frac{d_0}{d}\right),$$

so that extension of the lever arm due to this cause cancels out.

The ratio $\frac{d_0}{d}$, the relative density of $CuSO_4$, to copper will only alter very slightly with temperature, and is negligible.

Capillarity.—This will add a small force tending to prevent the plate rising. As the edges of the plate are small compared with its breadth it will merely add a term to be balanced by W, viz.

$$2\beta T \cos a,$$

where β is the breadth, T surface tension in dynes per cm. length, a the angle between this force and the vertical owing to the meniscus.

Evaporation.—This might cause a change of level of the liquid altering the amount of connecting wire immersed.

Creeping.—Generally with $CuSO_4$ solutions, there is a tendency for a deposit to take place on portions of the plate out of the liquid. This would tend to upset the equilibrium if the meter is left unattended for a long period.

Bastian Meter

This meter depends on the decomposition of water and measures

$$\int_0^t \mathrm{I}dt.$$

The electrodes ordinarily consist of iron concentric cylinders immersed in a solution of sodium hydrate. In the earlier types of meters, the electrodes were of platinum, and the solution used was dilute sulphuric acid.

Some oil is poured on the top of the solution to prevent evaporation, and also to act as an index.

It is a series meter and is acted upon by the total current passing. As will be seen from Fig. 82, the action depends on the fact that hydrogen and oxygen are liberated and, consequently, the level of the liquid falls. The amount decomposed is assumed to represent Board of Trade units.

Owing to the polarisation E.M.F., which in this case for water and bright platinum electrodes in dilute sulphuric acid is as much as 1·07 volts, and an electro-

motive force of 1·7 volts is required to continually decompose a solution, this appears a serious objection, especially when the IR drop due to any current has to be added. Some of these meters have a volt drop as much as 3 volts (*vide* "Electricity Meters," Ratcliffe and Moore, *Proceedings I.E.E.* vol. xlvii. p. 3) at full load which would necessarily affect the candle power of lamps on a 100-volt circuit. With platinum black electrodes, the decomposition E.M.F. drops to 1·07, and any E.M.F. above this will cause decomposition. We may, therefore, assume in this meter that the polarisation E.M.F. is at least a volt, and to this must be added the IR drop.

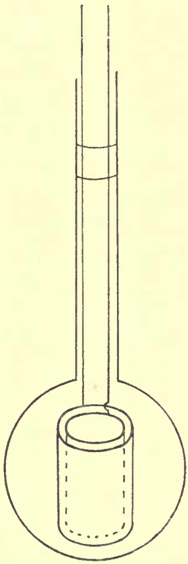


FIG. 82.—Bastian Meter.

When the current is passing through the meter owing to the evolution of gases it is impossible to make an accurate reading. Since a coulomb liberates 0·000010384 and 0·00008286 grms. of hydrogen and oxygen respectively, we see that 0·000093 grms. of water are decomposed by the passage of a coulomb.

In one hour therefore

$$0{\cdot}000093 \times 3600 \text{ grms. are decomposed,}$$
$$= 0{\cdot}3348 \text{ grms.}$$

If the meter was working on a 100 volt circuit, this would represent the decomposition due to the use of one Board of Trade unit.

Let v be the volume of the tube containing the solution, then

$$v_1 = 0{\cdot}79\ \mathrm{D}^2\mathrm{L}.$$

The volume decomposed is

$$v_2 = 0{\cdot}78\ \mathrm{D}^2 x,$$

D being the diameter of the tube and L the length. If x is the distance from zero, then

$$\frac{x}{\mathrm{L}} = \frac{v_2}{v_1}.$$

As the tube in some of these meters is about 5 cms. diameter and length of scale about 20 cms., we see that the reading

$$x = \frac{{\cdot}3348 \times 2}{78},$$

$$x = 0{\cdot}086 \text{ cm. approximately,}$$

or 10 units only represent a length of scale of 0·43 cm. It is certainly quite impossible to read the scale correctly to half a millimetre, and yet the graduations are of about this amount.

To make up for the decomposed water, fresh water must be added from time to time.

Consequently in such meters, all record of the previous consumption is obliterated, this being common, however, on all electrolytic meters at present.

Although a great many objections to this type of meter have been raised on grounds of back E.M.F. errors in reading general "messiness," the fact remains that for small installations of an ampere or so, such meters have an advantage. They are simple, repairs are cheap, and initial cost is very low. They certainly register the smallest current correctly, and are independent of

temperature; in fact, a rise of temperature in working diminishes the internal resistance of the meter. They are also independent of all fluctuations of load.

If on high loads the back E.M.F. and resistance cause an excessive drop, it must be remembered that this is generally only for a short period, and taking ordinary fluctuations of voltage on the mains into account, we are of opinion that far too much importance is attached to this question of volt drop, especially on 200 or higher voltage circuits.

Supply Meters

Experiments with Shunted Electrolytic Meter.—The following experiments were made with an electrolytic meter in order to test the value of the back E.M.F. and the resistance.

Test I.—

M represents the meter unshunted.
A represents movement of an ammeter, resistance 1·00 ohm.
G, Galvanometer.
B, Battery.

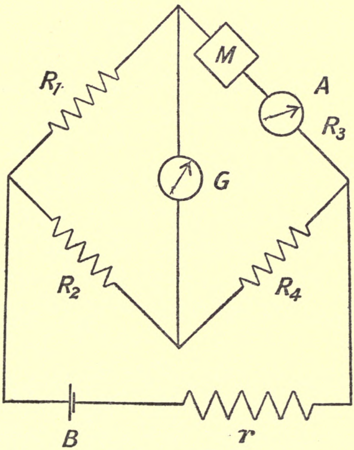


Fig. 83.—Test I. on shunted electrolytic meter.

The current was read on the ammeter A and adjusted by the resistance *r*. The results were as follows :

Divisions.	Amperes.	R_4.	Remarks.
100	·027	47·82	R_4 read to 0·1 ohm 2nd decimal place obtained by calculation from galvanometer deflection.
75	·02025	47·88	
50	·0135	47·90	
20	·0054	48·10	

If we call R_3 the resistance of the bridge arm containing the meter, e the supposed back E.M.F. of the meter, i the current in the meter when balance is obtained, then we have

$$iR_3 + e = iR_4, \text{ if } R_1 = R_2,$$

$$\therefore \quad i \text{ in } R_1 = i \text{ in } R_2.$$

If the back E.M.F. vary with i we have for different currents i_1, i_2, E.M.F.'s e_1, e_2.

Let
$$e_1 = i_1(R_4 - R_3),$$
$$e_2 = i_2(R'_4 - R_3),$$
$$i_1 = 2i_2,$$

then
$$\frac{e_1}{e_2} = 2\frac{(R_4 - R_3)}{(R'_4 - R_3)}.$$

Since
$$e_1/e_2 < 2,$$

hence
$$\frac{R_4 - R_3}{R'_4 - R_3} < 1, \text{ or } R_4 < R'_4.$$

As the value of the current is increased, the value

of R_4 diminishes. This at first might appear strange; it is due to the fact that e increases less rapidly than i.

Employing the values for "e" found in later test, we have:

Current.	e.	$R_4 - R_3$.	R_3.	R_4.
·027	·00766	·284	47·536	47·82
·02025	·00628	·310	47·570	47·88
·0135	·0049	·363	47·557	47·90
·0054	·00277	·513	47·587	48·10

$$R_4 - R_3 = \frac{e}{i} = \frac{\cdot 00766}{\cdot 027} = \cdot 284 \left\{ \begin{array}{l} \text{Mean value of } R_3 = 47{\cdot}558. \\ \quad \text{Checked by Kohlrausch} \\ \quad \text{method, } 47{\cdot}56. \end{array} \right.$$

Values of R_3 above differ from mean by 1 in 1000 at most, which is within error of instruments.

The value of $R_4 - R_3$ is given in the column (see above), and the actual value average of R_3 gives a very close agreement, viz. R_3 mean $= 47{\cdot}558$ ohms. This confirms the fact that the microammeter readings are a measure of the back E.M.F.

Test II.—Effect of Tilting or Shaking the Meter.—The same connections were used as above, but a microammeter substituted for the galvanometer. It was balanced for 100 divisions 0·027 amperes. Tilting and shaking produced a deflection of 10 divisions on microammeter showing a want of balance.

To prove that this was not due in any way to contact resistance altering, the current was again brought to 100 divisions, and balance after a little time was restored.

Test III.—Back E.M.F.—The arrangement was

altered so that the "meter" discharged through a total resistance of 46·6 + 60 ohms. The former was the

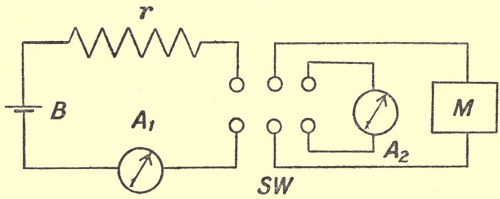


FIG. 84.—Connections of Test III.

A_1, Ammeter movement. M, Meter. B, Cell. *r*, Resistance.
A_2, Microammeter. SW, Two-pole, two-way switch.

resistance of the electrolyte and the latter that of the microammeter.

The following curve (*a*) was obtained:

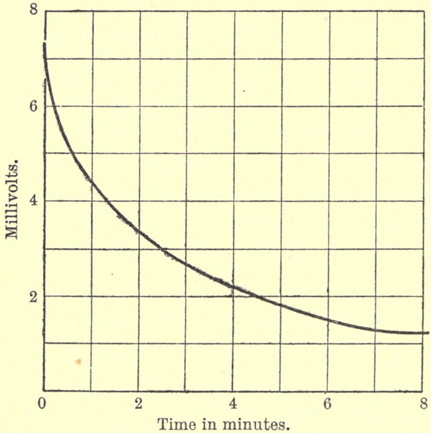


FIG. 85.—Discharge curve; microammeter permanently in circuit.

Curve of Results.—(*b*) The cell was then charged, and short circuited through a switch, the milliammeter form-

ing a shunt. At intervals the switch was opened and the milliammeter read. The results were as shown below:

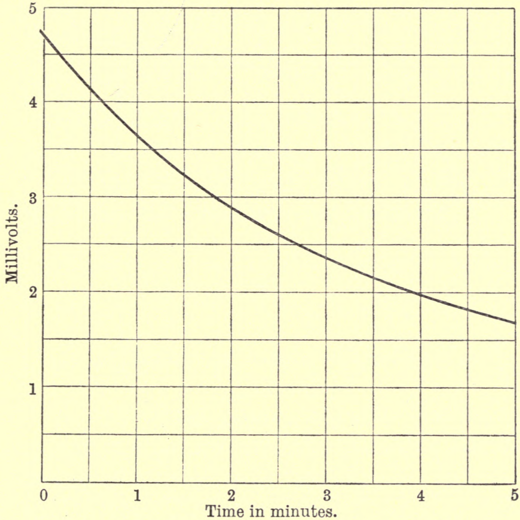


FIG. 86.—Discharge curve; cell short circuited.

Curve.—(*c*) It is seen that the results are unaffected by doubling the resistance in circuit.

Another test was taken of the meter on open circuit. The values obtained were initial deflection 4·74, final 1·7 millivolts, time 300 seconds.

Apparently, then, the running down is independent of resistance.

Calculation shows this effect not due to thermal E.M.F.'s. The total heating $I^2r = \cdot 036$ approximately, and the microvolts were 7660 as maximum. Even assuming 50 microvolts per 1° C., the temperature would require to be about 130° C., which is out of the question.

The following curve illustrates the charging effect:

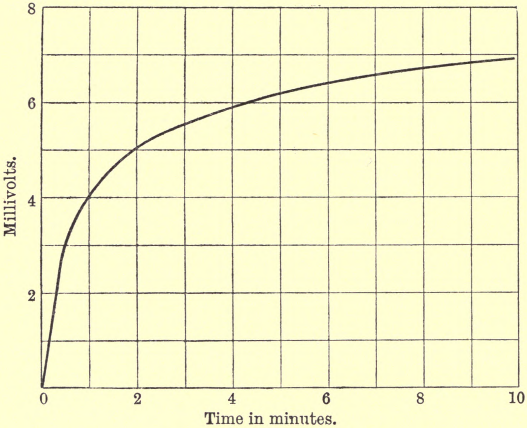


FIG. 87.—Charging curve.

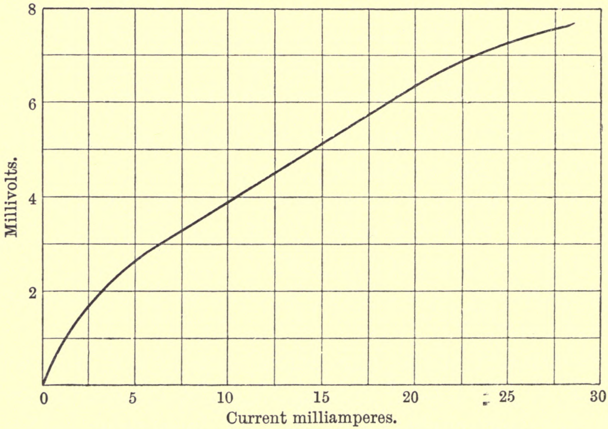


FIG. 88.—Relation between current and B.E.M.F.

current was 0·027 amperes. It was charged for half a minute and reading noted, then allowed to discharge,

then charged for one minute and so on. The relation between current and back E.M.F. is shown in Fig. 88.

MOTOR METERS—I.

Permanent Magnet.—No shunt.

In motor meters of this type—such as Chamberlain and Hookham and Ferranti newer type—with eddy currents brake disc, we have the following:

$$\text{Driving torque } T = \frac{ZFC}{20\pi},$$

$$\text{Retarding torque} = k_1 n + k_2,$$

where k_1 and k_2 are constants, the latter being mechanical friction, and $k_1 n$ the eddy current braking effect.

Consequently, for steady running

$$\frac{ZFC}{20\pi} = k_1 n + k_2.$$

By making k_2 small and F as strong as possible, the latter term is rendered negligible, and so

$$C = Kn,$$

where K is a constant.

Compensating Coil.—Owing to fluid friction of mercury at the higher speeds, the speed of the motor is too low to register correctly, and hence a series compensating coil is used to weaken the field acting on the brake disc and so increase the speeds by the right amount.

The following curves are the results of a test on a

Chamberlain and Hookham meter with and without the compensating coil:

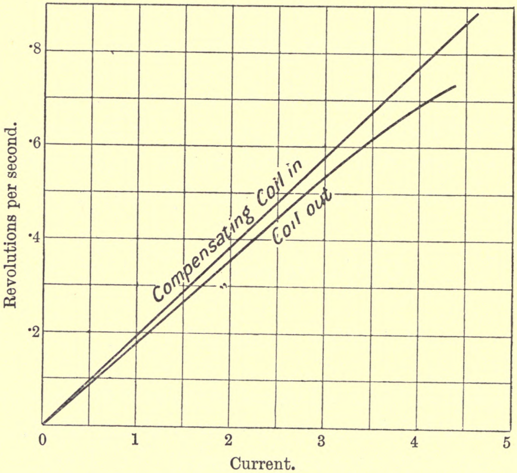


FIG. 89.—Test on Chamberlain and Hookham Meter. Effect of compensating coil.

Coil in.			Coil out.		
Current.	Revs.	R.P.S.	Current.	Revs.	R.P.S.
2·93	67	·552	3·35	71	·592
3·75	87	·726	4·1	84·5	·705

In all cases 2 minutes' readings were taken.

Testing constant, 5·14 seconds per rev. per ampere. Checking the last two readings:

Coil in.

Current 3·75, hence speed should have been

$$\frac{3{\cdot}75}{5{\cdot}14} = {\cdot}730.$$

Actual, ·726 meter reads ca. 0·5 per cent low.

Coil out.

Current 4·1 {Calculated speed . ·797
Actual speed . ·730

Difference . ·067
ca. 8½ per cent low.

Motor Meters—II.

Permanent Magnet.—Shunted armature. Retarding torque constant. In this case if e is the back E.M.F. of the armature, we have

$$e + ir = (\mathrm{I} - i)\mathrm{R}.$$

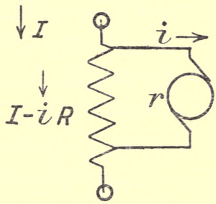


Fig. 90.

If the retarding torque be assumed constant, then that is equivalent to assuming i constant.

Putting for e the value,

$$k_1\mathrm{BN},$$

where k_1 is some constant, B the flux density and N the revolutions per second. Hence

$$\mathrm{N} = \frac{\mathrm{IR}}{k_1\mathrm{B}} - \frac{i(\mathrm{R}+r)}{k_1\mathrm{B}}.$$

Also since the driving torque is proportional to Bi, if $Bi=k_2$ the friction torque, then

$$i=\frac{k_2}{B},$$

so that
$$N=\frac{IR}{k_1B}-\frac{k_2(R+r)}{k_1B^2}.$$

Hence we see that in such meters the error $\frac{k_2(R+r)}{k_1B^2}$ can be diminished by diminishing R and r, also if B is doubled the error will be only a quarter. For accuracy, therefore, they should be worked with as high a permanent magnetisation as possible.

If the latter term is negligible, then

$$N \propto I,$$

and the meter acts as a coulomb meter.

The ratio of error to meter reading can be obtained from the expressions above. For small values of I the error may be considerable.

Some meters of the O.K. type, tested by Mr. Evershed (see *Journal of I.E.E.* vol. xlvii. p. 69), gave the following results :

Load.	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{2}$	Full.	
Meter A . .	14·5	3	0·7	0	Per cent error.
Meter B . .	10	3·2	2	0	

The following curve is taken from a paper, " Elec-

tricity Meters and Notes on Meter Testing," by Messrs. Ratcliffe and Moore, *I.E.E.* vol. xlvii. p. 3 :

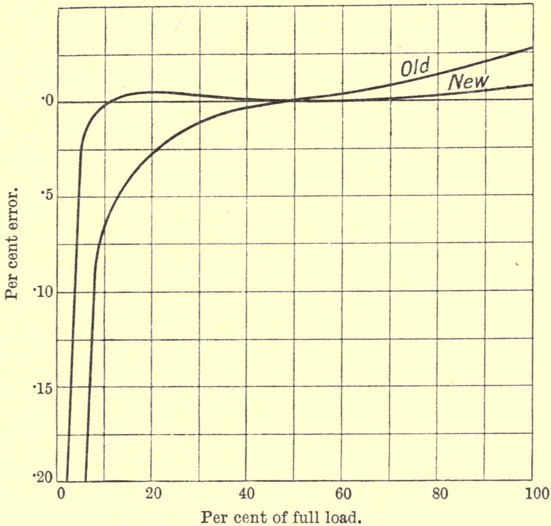


FIG. 91.—Feston Thomson Commentator Meter.

As the armature windings are wound on metal formers which have eddy currents induced in them, this makes a difference in the retarding torque.

The following are the results of a test of a British Thomson Houston O.K. meter by the authors :

[TABLE

METER STATIONARY

E.	*i.*
Volts.	Divisions.
·022	12
·04	22
·08	45
·16	84
·188	100

Slight variation in resistance according to position of armature.

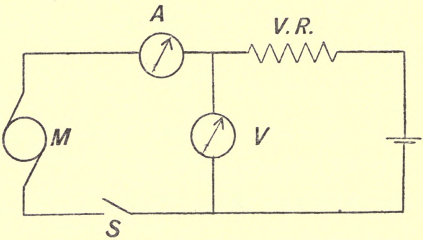


FIG. 92.—Test on B.T.H. O.K. Meter.

M, Meter armature (shunt disconnected).
A, Movement of moving coil ammeter.
·00027 amps. per div.—150 divs.
1·00 ohm resistance.
V, 1·2 volts range—120 divs.
Res. 106 ohms—moving coil.
S, Switch on meter.

Mean values given above.

R. of armature, leads, switch, and ammeter.

$$\frac{\cdot 188}{\cdot 027} = 6\cdot 97 \text{ ohms.}$$

R. of leads and switch, ·47 ohms.
R. of ammeter, 1·00 ohm.
R. of armature, $6{\cdot}97 - 1{\cdot}47 = 5{\cdot}5$ ohms.

$$\text{R. of shunt } \frac{\cdot 18}{\cdot 89} = \cdot 202 \text{ ohms.}$$

METER RUNNING—NO SHUNT

(Same connections as previously)

E.	C.	Time.	Dial Reading.	Revs.	R.P.S.
Volts.	Divs.	Secs.	Units.		
·175	5	46	·001	50	1·085
·36	8·5	38·8	·002	100	2·58
·415	10	96·6	·006	300	2·90
·56	13	75·8	·006	300	3·96
·68	16	64·8	·006	300	4·53

Though meter only has 4 part commutator, readings fairly steady.

At max., reading ammeter quite steady. Voltmeter pointer—slight flicker—ca. 1/3 div.—mean taken.

$$i \text{ (divs.)} = 1{\cdot}2 + 21 \text{ volts,}$$
$$i \text{ (amperes)} = \cdot 00032 + \cdot 00567 \times \text{volts,}$$
$$\text{R.P.S.} = 6{\cdot}85 \times \text{volts.}$$

Volts across terminals . . .	1·00
Current	·006
R.P.S.	6·85

Drop in ammeter, leads and shunt, $\cdot 006 \times 1{\cdot}47 = \cdot 0088$.

Drop in armature, $\cdot006 \times 5\cdot5 = \cdot033$.

Total drop, $\cdot033 + \cdot0088$.

B.E.M.F. $= \cdot9522$.

Motor terminal volts $= \cdot9912$.

$$\text{B.E.M.F.} = \frac{\cdot952}{\cdot991} \text{ motor terminal volts.}$$

$$= \cdot960 \text{ motor terminal volts.}$$

$$\text{Armature drop} = \frac{\cdot033}{\cdot991} = \cdot0333 \text{ motor terminal volts.}$$

i.e., armature drop is 3·3 per cent of motor volts and is quite appreciable.

$$\text{Speed} = \frac{6\cdot85}{\cdot991} \text{ motor terminal volts.}$$

$$= 6\cdot92 \text{ motor terminal volts.}$$

Assume a current of 2·5 amps. in installation.

Drop across shunt $2\cdot5 \times \cdot202$.

$= \cdot505$ volts.

Speed $= 3\cdot51$ R.P.S.

Revs. per hour, $3\cdot51 \times 60 \times 60$.

Units recorded per hour,

$$\frac{3\cdot51 \times 60 \times 60}{50 \times 1000} = \cdot2525.$$

Time units, ·25.

Error, 1 per cent high.

Considering the expression

$$e = (\mathrm{I} - i)\mathrm{R} - ir,$$

when $\mathrm{I} = 2\cdot5$,

$$i = \cdot00318,$$

though we may say that $I - i$ is practically equal to C, the term ir is not negligibly small, and if i were constant and not nearly proportional to speed, the error introduced would be appreciable.

Running down tests.

Connections as in Tests I., II.

The meter (unshunted) was run at a constant speed, this speed being determined by the readings of the voltmeter connected across its terminals.

The switch was opened and the time taken to come to resistance noted.

The test was repeated with various initial speeds. It was found impossible to take a set of readings in the usual way on the voltmeter, as the meter slowed up on account of the damping effect of the currents induced (R. of voltmeter, 106 ohms, and when meter generates ·7 volts, current ·07 amperes approximately, considerably more than the running C. of the meter). For the same reason the shunt had to be disconnected.

Terminal Volts.	Time to come to Rest in Secs.
·785	116
·59	103
·48	93·4
·405	85·0
·345	76·0
·29	70·0
·18	49·0

By plotting initial speed $\propto$ volts and time, the following table was obtained:

Time in Secs. from Start.	E.	Time in Secs. from Start.	E.	
0	·72	60	·188	
10	·58	70	·145	
20	·47	80	·110	extrapolsted.
30	·38	90	·08	
40	·305	100	·055	
50	·24			

The starting-point was taken as 72, as it came conveniently on the curve.

Losses at a mean speed $\frac{n_1+n_2}{2}$ are proportional to

$$\frac{n_1^2-n_2^2}{t}=k\frac{n_1^2-n_2^2}{t},$$

and since

$$n \propto \mathrm{E},$$

$$\text{Losses}=k'\frac{\mathrm{E}_1^2-\mathrm{E}_2^2}{t}.$$

E_1.	E_2.	$\frac{\mathrm{E}_1+\mathrm{E}_2}{2}$.	$\mathrm{E}_1^2-\mathrm{E}_2^2$.
·72	·58	·65	·182
·58	·47	·525	·1155
·47	·38	·425	·0765
·38	·305	·343	·0514
·305	·24	·273	·0355
·24	·188	·214	·0225
·188	·145	·161	·0137
·145	·110	·123	·0089
·110	·08	·095	·0057
·08	·055	·068	·0034

Relation between $E_1^2 - E_2^2$ and $\frac{E_1 + E_2}{2}$ is of the form

$$E_1^2 - E_2^2 = \frac{E_1 + E_2}{100} + \frac{(E_1 + E_2)^2}{10},$$

i.e. losses are partly proportional to speed and partly to square of speed.

When $$E = \cdot 65,$$

$$\text{Speed} = 6 \cdot 85 \times \cdot 65 \text{ R.P.S.} = 4 \cdot 46 \text{ R.P.S.},$$

$$\text{B.E.M.F.} = \cdot 65 \times \cdot 952 = \cdot 619,$$

$$i = \cdot 00405,$$

$$\text{Losses} = k\frac{E_1^2 - E_2^2}{t} = k\frac{\cdot 182}{10} = \cdot 002476,$$

$$k = \frac{\cdot 00248 \times 10}{\cdot 182} = \frac{\cdot 00248}{\cdot 182} = \cdot 1363.$$

Losses at any speed N (the mean of n_1 and n_2)

$$= \cdot 01363 \left\{ \left(\frac{n_1 + n_2}{6 \cdot 85 \times 100} \right) + \left(\frac{n_1 + n_2}{6 \cdot 85} \right)^2 \frac{1}{10} \right\},$$

$$= \cdot 01363 \left\{ \left(\frac{2N}{6 \cdot 85 \times 100} \right) + \frac{4}{6 \cdot 85^2} \frac{N^2}{10} \right\},$$

$$= \cdot 00004N + (\cdot 000116)N^2.$$

Trying $$N = 4 \cdot 46 \text{ (as above)},$$

$$\text{Losses} = \cdot 0001784 + \cdot 00231,$$

$$= \cdot 002488 \text{ (as before)}.$$

It appears therefore that the retarding torque in an O.K. meter is not constant, but is partly due to n, and partly to n^2, and the latter is more important.

Shunted Ampere Hour Meter with retarding torque varying as the speed.

In this case $e+i(r+R)=IR$, and $e=k_2n$, and $i=k_3n$,

$$k_2n+k_3n(r+R)=IR,$$

$$\therefore \quad n=I\frac{R}{k_2+k_3(R+r)}.$$

If we write $k_2=AB$ and $k_3=C/B$ where B is the magnetic induction and A and C are constants, we see that

$$n=I\frac{R}{AB+\frac{C}{B}(R+r)}.$$

We see that an increase in B would in general diminish the speed, since we may write above thus

$$n=I\frac{R}{B\left\{A+\frac{C}{B^2}(R+r)\right\}}.$$

An increase in B also tends to diminish the term $\frac{C(R+r)}{B^2}$ rapidly.

Mercury Meter

Ferranti Type.—This is one of the oldest and best known types of electricity meters.

As will be seen from Fig. 93, this meter originally

consisted of a vane revolving in a flat box arranged horizontally between the poles of a powerful electro-magnet. The current flows radially from the centre to the outside of the mercury trough. The rotation of the vane drove the counting mechanism, and by suitable braking the speed was made proportional to current.

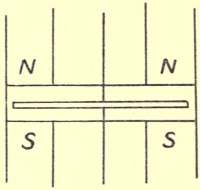


FIG. 93.—Ferranti Meter.

It is shown (*vide Hydrodynamics*, by H. Lamb, p. 27) that when a current flows in this way, the angular velocity of the mercury

$$\omega = \frac{\lambda}{r^2}$$

and $\lambda = \mu t$ where

$$\mu = \frac{ZI}{2\pi\rho},$$

Z being the strength of field perpendicular to the mercury, I the total current, ρ the density of the liquid.

$$\therefore \quad \omega = \frac{ZIt}{2\pi\rho r^2},$$

or the angular velocity increases with time and varies inversely as the radius,

$$\therefore \quad \frac{d^2\theta}{dt^2} = \frac{ZI}{2\pi\rho r^2}.$$

If now the electromagnet is worked at a low induction, $Z \propto I$, and this can be made a linear relation within certain limits, hence $Z = kI$ where k is a constant,

$$\therefore \quad \frac{d^2\theta}{dt^2} = \frac{kI^2}{2\pi\rho r^2}.$$

Since for any rotating body,

$$\frac{d^2\theta}{dt^2}=\frac{\text{moment of forces}}{\text{moment of inertia}},$$

we see that the torque $\propto \mathrm{I}^2$, and in order that the acceleration may be zero, this must equal the retarding torque for the speed. Also the retarding torque must $\propto n^2$, the number of revolutions, in order that

$$\sqrt{\overline{\mathrm{I}^2}}=k\sqrt{\overline{n^2}}$$

or

$$\mathrm{I}=k_1 n.$$

This was accomplished by serrating the top and bottom of the flat box so as to obtain a square law for the fluid friction.

Consequently the meter revolved with a velocity proportional to the current and registered the quantity

$$\sqrt{\int i^2 dt}=\mathrm{N},$$

where N is the total number of revolutions.

This meter is only suitable for continuous currents, as iron magnets and the magnetic action on the mercury rotation becomes complex when the current is alternating.

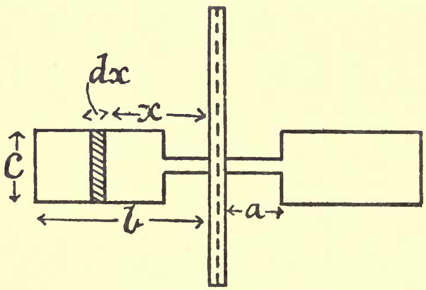


FIG. 94.—Torque acting on vane.

If the driving torque varies as shown above, then on an elementary portion cdx of a vane where x is the radius, the total torque will be the sum of the elementary torques, or

$$T = c\int_a^b \frac{dx}{x^2}$$

or

$$T = c\left(\frac{b-a}{ab}\right),$$

where c is some constant. By making the vane of suitable shape, sufficient torque could be obtained to work the train of wheels.

In the newer type of meter described below, the fluid friction of mercury causes the readings to be low for high speeds, and consequently some form of compensation is necessary.

With mercury meters following a true square law we have seen that

$$I^2 = kn^2.$$

Consequently, if I fluctuates

$$N = \sqrt{n_1^2 + n_2^2 + n_3^2 + \text{etc.}};$$

in other words, N reads the "root mean square value," instead of the arithmetical average value. In this respect a square law is unsatisfactory. In addition also to fluid friction, which changes for different speeds, there is always a certain amount of ordinary mechanical friction, which must be compensated.

See p. 292 regarding starting and stopping.

Elihu Thomson Meter

In this case the field is due to the main series coils, and the armature forms a shunt across the mains, together with a resistance and compensating coil.

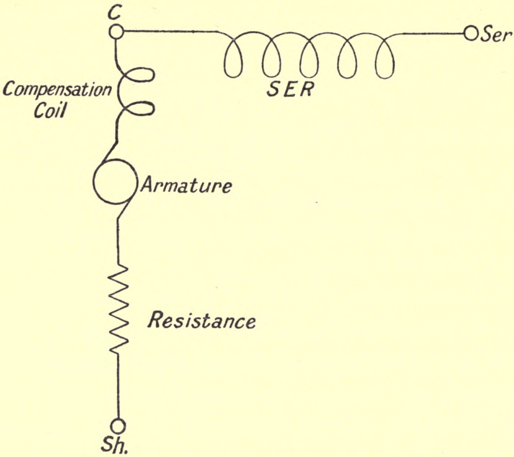


FIG. 95.—Thomson Watt-hour Meter.

Let i = shunt current in amperes,
I = main current in amperes,
Z = no. of armature conductors,
T = torque in dyne cms.,
R = total resistance of shunt circuit,
E = supply voltage,
f = flux magnetic due to i,
F = flux magnetic due to I,
n = revolutions per second.

The driving torque consists of two parts, viz. main coil torque and compensating torque.

$$T = \frac{ZFi}{20\pi} + \frac{Zfi}{20\pi},$$

and since the coils contain no iron

$$F = k_1 I \text{ and } f = k_2 i,$$

therefore $$\mathrm{T}=\frac{\mathrm{Z}}{20\pi}\mathrm{I}k_1 i+\frac{\mathrm{Z}}{20\pi}k_2 i^2.$$

Again,

The retarding torque = brake torque + friction torque.

Now in a brake disc the current induced by a magnetic impulse penetrates into the mass similarly to heat flow. They, therefore, follow the same laws as for heat, and regarding the mass as large

$$4\pi\mu\frac{di}{dt}=\sigma\frac{\partial^2 i}{\partial x^2}.$$

Hence if I_0 is the current on the surface, it will decay according to the law

$$\mathrm{I}=\mathrm{I}_0\epsilon^{-\frac{x^2\pi\mu}{\sigma t}}/\sqrt{t}$$

where x = depth,
μ = permeability,
σ = specific resistance,
t = time,

and since the initial E.M.F. causing it is proportional to $\frac{d\mathrm{N}}{dt}$, the lines of induction cut by the disc, we regard current as proportional to $\frac{d\mathrm{N}}{dt}$, and consequently braking effect proportional to n, the revolution per second, since the magnetic field is that due to permanent magnets acting on the brake disc.

Braking torque for eddies, therefore, $=k_3 n$.

Friction torque is generally regarded as a constant, say k_4.

Hence an equation for constant speed or $\dfrac{d^2\theta}{dt^2}=0$ gives

$$\frac{Z}{20\pi}Ik_1 i+\frac{Z}{20\pi}k_2 i^2=k_3 n+k_4.$$

If i is constant, the torque due to the compensating coil for a given value of i may, by suitably adjusting turns and position of coil, be made equal to k_4. Hence

$$\frac{Z}{20\pi}Ik_1 i=k_3 n.$$

Now $i=\dfrac{E-e}{R}$ when e is the back E.M.F. of the motor. If e is negligibly small, then $i=\dfrac{E}{R}$, and

$$\frac{Z}{20\pi}kIi=\frac{Z}{20\pi}\ \frac{kIE}{R}=k_3 n,$$

or $$n=IEk\frac{Z}{20\pi R},$$

$n=KIE$ where K is a constant, and the speed varies with the watts expended in the circuit.

Effect of e.—In the above investigation e is regarded as negligible. Since

$$T=FZi/20\pi,$$
$$e=FZn/10^8$$
or $$e=20\pi Tn/i\times 10^8.$$

Putting $T=8$ grm. cms., $i=0{\cdot}02$, $n=150$ R.P.M. or 2·5 R.P.S., $e=0{\cdot}46$. By actual test on a Siemens meter, 5 amp. 100 volt size, load current 4·4 amps., the following results were obtained:

Volts 100. Meter stationary $i = 52 \times \cdot 00027$.

$i = \cdot 01405$ amperes.

Meter rotating $i = 51 \cdot 8 \times \cdot 00027$.

$i = \cdot 01400$.

Total resistance in circuit (shunt) = 7100 ohms.

Resistance of armature = 604 ohms.

Armature drop (meter stationary) = 8·5 volts.

Meter rotating = 8·9.

Difference = 0·4 volts.

i was read on a moving coil ammeter, 100 divisions corresponding to ·027 amperes. The difference in the values of i are not due to contact resistance, shown by rotating the armature by hand series coils non-excited (in which case i and drop are constant), and comparing with the readings obtained with series coils excited. Both shunt and series windings were energised from separate sources (cells in both cases) so as to obtain steady readings and eliminate any effect due to drop on the mains on switching on the series circuit.

The maximum full load error due to e is therefore only about $\frac{1}{2}$ per cent low, the error being proportional to I.

Effect of Compensating Coil.—

Coil out of Circuit, Table I.

Coil in Circuit, Table II.

The value of i was kept constant at 52 divisions on instrument $= \cdot 0140_5$ amps.

[TABLE

(1) Coil out of Circuit

(Equivalent resistance substituted, ca. 2800 ohms.)

C.	Revs. of Arm.	Time.	R.P.S.	R.P.S.
		Secs.		Calculated.
·55	20	178	·112	·115
1·03	40	176	·227	·227
1·60	60	169·8	·354	·360
2·20	80	160	·500	·501
2·60	100	168	·595	·595
3·05	125	178	·702	·700
·20	6	194·4	$\cdot030_9$	·033
·30	10	180	$\cdot055_6$	·056
·15	Just creeps	...	...	...
·10	Stationary	...	...	...

(2) Coil in Circuit

(Value of i constant as before.)

C.	Revs. of Arm.	Time.	R.P.S.	R.P.S.
		Secs.		Calculated.
·10	Just creeps	...	...	...
·20	6	144	·042	·0468
·30	10	153	$\cdot065_4$	·07
·54	15	125	·120	·1265
1·15	40	148	·270	·269
1·70	60	153	·392	·398
2·18	80	156·8	·510	·510
2·60	100	163·2	·611	·609
3·07	125	173	·722	·720

The resistance in the load circuit was rather coarse, which accounts for values of I obtained—the ammeter

was calibrated on the potentiometer and found to be dead accurate.

Plotting load current (I) and R.P.S.

Curves obtained as shown:

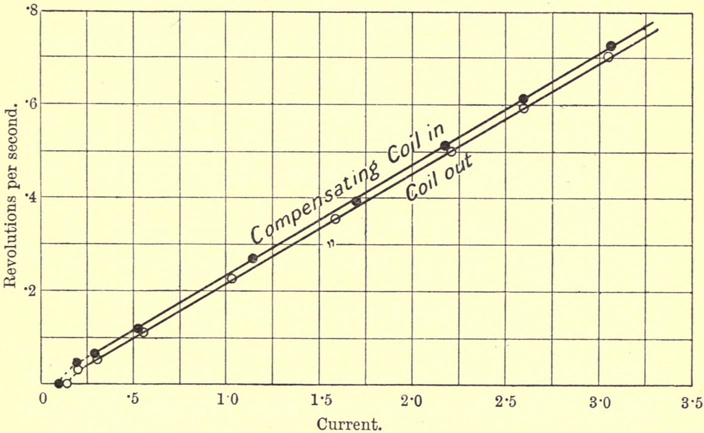


Fig. 96.—Calibration of motor watt-hour meter; voltage constant.

Higher values of I than 3 amps. were not taken, as the disc ran too quickly for the revolutions to be counted and the testing dials go too slowly.

Time taken on stop-watch reading to 1/5 sec.

Relation between speed and I (coil in):

$$\text{R.P.S.} = \cdot 234 \text{ current in amps.}$$

Speed and current (coil out):

$$\text{R.P.S.} = \cdot 234 \text{ current} - \cdot 014.$$

Results are seen from last column to be in fair agreement.

$$\text{Revs. of arm. per unit} = \cdot 234 \times \frac{1000}{100} \times 60 \times 60,$$

$$= 8424.$$

Makers' figure, 8430.

More correct number (from teeth on gear wheels), 8428·6.

Error 5 in 8400, say ·06 per cent.

N.B.—The accuracy is remarkable, mostly due to luck in drawing straight line, from which equation was computed.

Meter Brake Discs

So far as the authors are aware, little seems to be written about the action of the eddy currents present in brake discs; and previous to discussing the action in induction meters it is advisable to look closely into this question.

In the first place it is shown in treatises on Mathematical Physics that the resistance of a thin infinite lamina of thickness t, when the current is led into it by electrodes of circular section, of radius a and b, is

$$R_1 = \rho/2\pi t \log_\epsilon D^2/ab,$$

where ρ is the specific resistance, D the distance apart of the electrodes, the thickness being considered small (*vide* J. J. Thomson's *Elements of Electricity and Magnetism*).

The lines of flow in this case are circles:

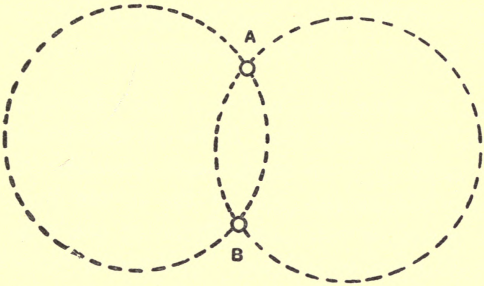


FIG. 97.—Lines of current flow ; electrodes on infinite lamina.

Again, in the same treatise, it is shown that if the electrodes A and B are placed on the periphery of a circular lamina thus :

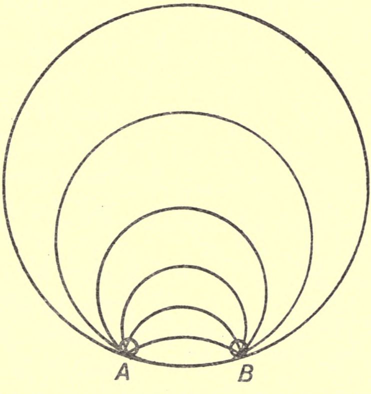


FIG. 98.—Lines of current flow ; electrodes near edge of disc.

the resistance will be

$$R_2 = \frac{\rho}{\pi t} \log_\epsilon D^2/ab.$$

Hence we see that

$$R_1 = \tfrac{1}{2}R_2,$$

provided D is the same and a, b, and t the same.

It will be noticed that if two electrodes are on a lamina, shifting them up towards the edge of it will practically reduce the path of the stream lines to one half its previous value, exactly as in the case of the circular disc just considered. This seems obvious from symmetry.

In such a case as that of two electrodes placed as in Fig. 99, it will be noticed that moving up towards the edge of disc still leaves a direct path for many of the stream lines, and only the curved ones are affected.

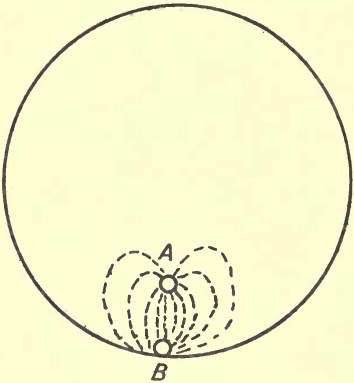


FIG. 99.—Lines of current flow ; electrodes placed along radius of disc.

Electrodes on Disc near Periphery.—In the case of eddy currents (see Fig. 101), it will be noticed that the currents will be more affected towards the periphery, and we should consequently expect a still greater resistance than double the central resistance in such a case.

If, then, eddy currents are generated by magnet poles in the usual way in a rotating disc, one would expect that, if the poles are well inside the disc, they will, provided there is no self-induction, approximate to the

condition for an infinite disc. Again, if the poles be moved near the circumference of the moving disc, the

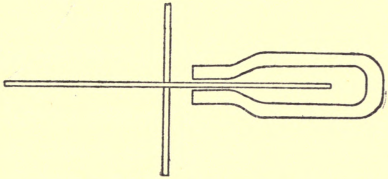


FIG. 100.—Poles well inside.

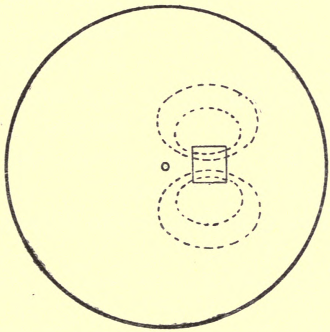


FIG. 101.—Poles well inside ; lines of current flow.

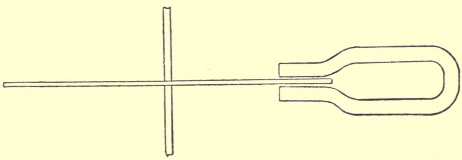


FIG. 102.—Poles near periphery.

resistance will increase. If there was no distortion of the stream lines, then from symmetry we might expect the resistance at the periphery to be more than double

that, when the poles are well inside the periphery. In this case the currents are generated radially.

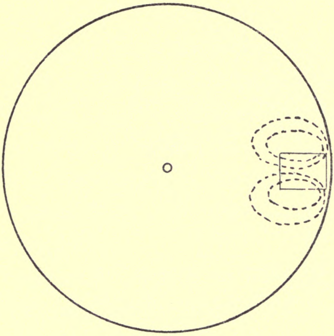


Fig. 103.—Poles near periphery ; lines of current flow.

However, it would appear in practice that as the pole is moved nearer to the periphery, the resistance increases very rapidly as the periphery is approached. This seems to arise from the fact that the currents are constrained to move in a comparatively small portion of the disc near the periphery, into which they are crowded, thereby greatly increasing the resistance.

An experiment was made in the following manner :

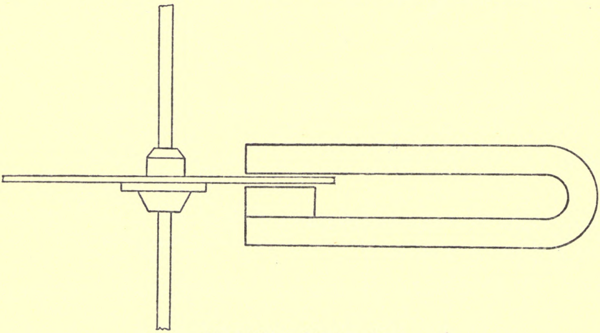


Fig. 104.—Brake disc.

A magnet as shown in Fig. 104 was moved along the radius. For each position the current was kept constant, and the time of a number of revolutions noted.

If we write the expression for torque as

$$T = \frac{kr^2\omega}{R} \qquad \text{(see p. 263)}$$

where ω is angular velocity, r is radius, R is resistance, then if the torque is kept constant by keeping the current constant through the meter, we have

$$R \propto r^2/t$$

where t is the time of a convenient number of revolutions.

In this way the curve 4 (Fig. 105) was obtained.

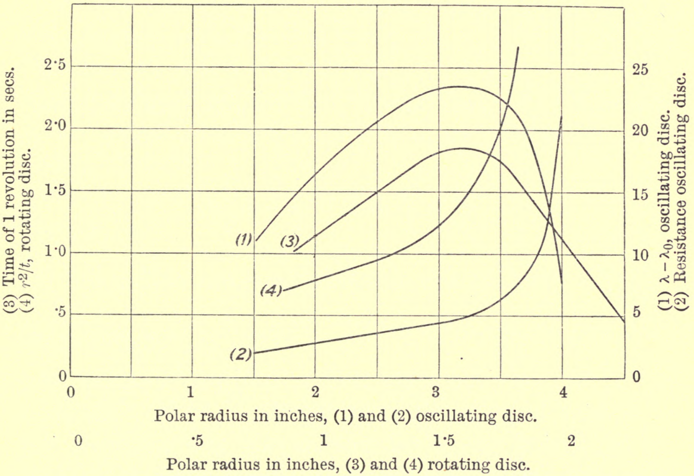


FIG. 105.—Curves showing relation between torque and resistance for various radii.

It will be noticed that if the resistance had continued increasing uniformly, it would be almost double that inside the disc at the periphery. In reality it increases much more rapidly towards the edge.

In another experiment a copper disc 8 inches in diameter was suspended by a brass wire 17·7 cms. long, and 0·03 cm. diameter perpendicular to its plane, the thickness of the disc being 0·076 cm. The logarithmic decrement in air was noted, and then the logarithmic decrements corresponding to the different radial positions of a magnet, when the disc was oscillated for these different positions.

Calling λ_0 the logarithmic decrement in air, λ the observed logarithmic decrement with magnet, r the radius to the magnet pole, then

$$\frac{r^2}{\lambda - \lambda_0} \propto \mathrm{R},$$

where R is the resistance. This follows from the equation of damping on p. 263, since it can be shown

$$\frac{\mathrm{K}_0}{\mathrm{I}} \propto \frac{\lambda_0}{\mathrm{T}} \text{ and } \frac{\mathrm{K}_0 + \mathrm{K}}{\mathrm{I}} \propto \frac{\lambda}{\mathrm{T}},$$

k is the damping due to electromagnetic action and involves $\frac{r^2}{\mathrm{R}}$ as shown on p. 263. Hence R varies as above.

In this way the curve 2 (Fig. 105) was obtained, the ordinates being numbers proportional to R. The time of oscillation was 17·05 seconds and was constant, the damping not being sufficient to alter it appreciably, and so making the above method of calculation possible. These results merely confirm the previous conclusion that in a metal disc (in latter case copper) the eddy current resistance increases rapidly towards the periphery of the disc.

Experiments also show that if a magnet is placed

so that the lines of force cut the disc twice, the torque for any position is exactly doubled, as is to be expected.

We see, therefore, that owing to the fact that the resistance increases in this way, it does not follow that the torque will increase on moving the brake magnet outwards towards the periphery, as would at first be expected.

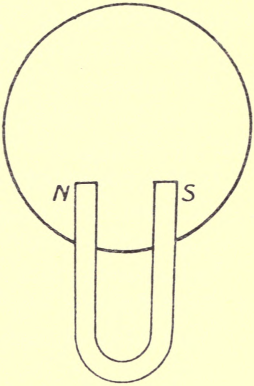


Fig. 106.

Torque Conditions.—If R was determinate, then if H is flux density, e the length of a filament of current acted upon, ω the angular velocity, or the radial distance of the pole from the centre, R the resistance of the disc, T the torque, then

$$T \propto \frac{Hr^2\omega}{R},$$

since e and r are assumed.

If $$\omega = 2\pi n = \frac{2\pi}{t},$$

then $$T = kr^2/Rt$$

for the different radial positions.

In the meter disc already referred to, the maximum torque occurred at a radial distance from the centre of 0·78 r. By observing t and keeping torque constant, the torque curve 3 shown above (Fig. 105) was obtained.

By means of the oscillation method with the disc already referred to these results were confirmed. The maximum torque occurred at about 0·812 r, and is shown in the torque curve above (1, Fig. 105).

These experiments seem to indicate that (*a*) there is a rapid increase of resistance towards the periphery of a damping disc; (*b*) that this resistance reduces the torque correspondingly.

It does not appear that the point of maximum damping is taken advantage of in meter construction, or in electric brake discs.

In meters the motion of the disc is very slow, whereas in electric brakes it is rapid.

The theory of meters will be discussed in the next sections.

The Induction Meter

This meter, as used for measuring A.C. electric energy, is by far the most difficult and complex.

Here a series electromagnet acts upon a disc inducing eddy currents in it, a shunt electromagnet also acts upon the same disc also inducing eddy currents. The general statement made about it is that the series magnet acts upon the shunt eddy and *vice versa*, the result being a torque driving the disc. To brake it permanent magnets are used, and these again induce eddies.

Supposing, therefore, we have a disc arranged as in Fig. 107, we shall endeavour to obtain a theory of the action.

Now it is clear that the flux from the series magnet is due to the magnetising component of the series current.

The flux due to the shunt magnet is proportional to the shunt volts. We shall denote the series flux by ϕ_s and

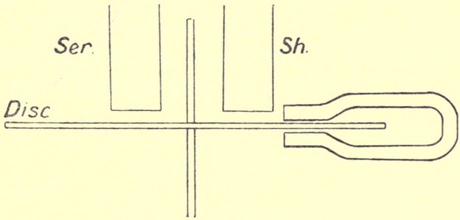


FIG. 107.—Diagram of induction watt-hour meter.

shunt by ϕ_{sh}. We shall assume the eddies are generated by these fluxes in the disc, thus :

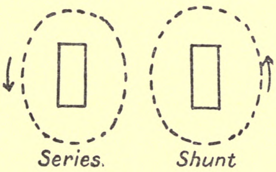


FIG. 108.—Eddies produced by shunt and series fluxes.

Now the series flux acts upon the filaments of eddy induced by it, but if we assume there is no self-induction in the disc then the mean torque throughout a period vanishes. The shunt flux, which is nearly 90° out of phase with the line voltage, also produces no torque on its own eddy if the same assumption is made. Next the series flux acts upon the shunt eddy producing a driving torque, and the shunt flux acts upon the series eddy producing a torque in the opposite direction. Hence the driving torque is a differential one, and under certain circumstances the meter may reverse its rotation.

Regarding the braking torque we may take this as proportional to the speed of the brake disc.

It may be mentioned that the magnetic flux causing eddies in the disc varies with the depth, and if H is the flux at any depth, H_0 the flux at the surface of disc, then for a depth small compared with the thickness,

$$H = H_0 \epsilon^{-md} \cos(\omega t - md),$$

where
$$m = \frac{\pi}{20}\sqrt{2\pi n},$$

where n is $2\pi \times$ frequency and d the depth (*vide* Russell, *Alternating Currents*, vol. i. p. 367). Since, however, neither the poles nor the eddies are very amenable to mathematical treatment, we shall merely assume that the flux in the disc is of the form $\phi = \phi_0 \cos nt$, where ϕ_0 is the maximum value of the flux and n the frequency.

Consequently, we have the following:

$$\text{Series flux} \propto \phi_s \cos(nt - \beta),$$
$$\text{Shunt flux} \propto \phi_{sh} \cos(nt - a),$$

and

$$\text{Series eddy} \propto -n\phi_s \sin(nt - \beta),$$
$$\text{Shunt eddy} \propto -n\phi_{sh} \sin(nt - a),$$

by differentiation.

Let t_1, t_2 represent the instantaneous torques due to action of series flux on shunt eddy, and shunt flux on series eddy, then

$$t_1 \propto n\phi_s \,.\, \phi_{sh} \,.\, \sin(nt - a)\cos(nt - \beta),$$
$$t_2 \propto n\phi_s \,.\, \phi_{sh} \,.\, \cos(nt - a)\sin(nt - \beta),$$
$$\therefore \quad t_1 - t_2 \propto n\phi_s \,.\, \phi_{sh} \sin(a - \beta).$$

The effect of the series, and shunt fluxes, on their own eddies integrates $= 0$ over a period.

If we substitute mean values taken over a period, the mean driving torque would be

$$T = nZ_s Z_{sh} \sin(\alpha - \beta),$$

$Z_s Z_{sh}$ being the mean fluxes, and if λ be the angle of lag between series current and supply volts, then

$$\alpha - \beta = 90 - \lambda,$$
$$\sin(\alpha - \beta) = \cos\lambda,$$

and we have

$$T = nZ_s Z_{sh} \cos\lambda.$$

Now, $nZ_{sh} \propto$ E the terminal volts on the shunt neglecting *cr* drop, and as is shown below, we may assume that C the series current $\propto Z_s$. Hence

$$T \propto EC \cos\lambda.$$

But the braking torque is kN where N are the revolutions per second. Hence

$$N \propto EC \cos\lambda.*$$

Series Coil.—Proportionality of C to Flux. C is the total current. Let i be the magnetising current and n periodicity, k a constant, then

$$C = \sqrt{i^2 + (kni)^2},$$

or vectorially.

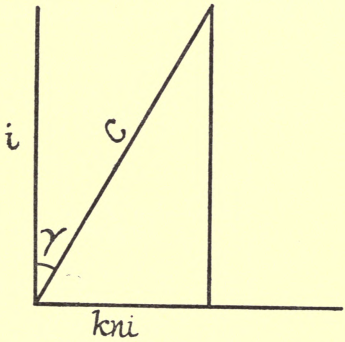


FIG. 109.—Vector diagram of magnetising and load current in series coil of induction meter.

Hence the volt drop will be $2\pi nLi$, 90° in advance of i, iR in phase with i,

* $\beta = \lambda + \gamma$, hence $\alpha - \beta = \alpha + \lambda - \gamma = 90 - \lambda$. Since λ is small, this is very nearly $\alpha - 90°$.

and kniR in phase with kni, or leading i by 90°. Hence the volt drop

$$v = i\sqrt{(2\pi n\mathrm{L} + kn\mathrm{R})^2 + \mathrm{R}^2}.$$

Hence if we write

$$\mathrm{C} = i\sqrt{1 + (kn)^2} \quad . \quad . \quad . \quad \text{(i.)}$$

$$\frac{v}{\mathrm{C}} = \frac{\sqrt{\mathrm{R}^2 + n^2(2\pi\mathrm{L} + k\mathrm{R})^2}}{\sqrt{1 + (kn)^2}} \quad . \quad . \quad \text{(ii.)}$$

Put $2\pi n\mathrm{L} + \mathrm{R} = a$, $k^2 = b$, then if v is observed for different frequencies, the values of a, b, and R may be calculated.

From (i.) above we see that the ratio $\frac{i}{\mathrm{C}}$ = constant for a given periodicity, so that we can say the series flux will be proportional to C.

A test was made as follows :

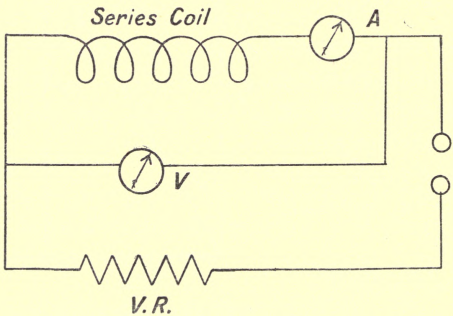


FIG. 110.—Determination of voltage across series coil of induction meter at various periodicities.

The series coil of an induction meter was put in circuit with some load, an ammeter and a voltmeter were con-

nected across its terminals, the ammeter volt drop being negligible. Frequency was read on a Hartman vibrating reed frequency meter. The volt drop for the series coil was read on a Paul unipivot dynamometer instrument.

Results

Current.	Periodicity.	Voltmeter Reading.	Volts.	Remarks.
3	0	77·5	1·385	Direct current.
...	20	82·3	1·466	
...	30	85·5	1·511	
...	40	89·2	1·575	
...	50	92·0	1·656	
...	60	97·0	1·745	
...	70	102·0	1·870	

From the table when

$$n = 30, \qquad E = 1{\cdot}511,$$

$$n = 60, \qquad E = 1{\cdot}745,$$

substituting in (ii.) above we obtain $R = \frac{1{\cdot}385}{3}$ ·

Solving $\qquad R^2 = 0{\cdot}2133,$

$$a = {\cdot}0000767,$$

$$b = {\cdot}0001206,$$

substituting in (ii.), and making n successively 20, 40, 50, 70, we obtain the following :

n.	E (calculated).	E (observed).
20	1·455	1·466
40	1·560	1·575
50	1·662	1·656
70	1·875	1·870

These results agree very well and justify the assumption made regarding the proportionality of series flux and current. The magnetising current

$$i = \frac{C}{\sqrt{1 + k^2 n^2}}.$$

If $C = 3$ amperes, the following gives values of i:

Periodicity.	i.
20	2·93
30	2·85
40	2·75
50	2·63
60	2·51
70	2·38

hysteresis being neglected.

Shunt Magnet.—If ϕ is maximum flux assume for sine wave

$$e = 4{\cdot}44 n\phi N/10^8,$$

where N are the turns on the coil. The terminal volts

$$E = e + IR \text{ vectorially,}$$

where IR is the ohmic volt drop, hence $E = e$ only when IR is negligible. IR equals 10 per cent in the case of

meters A and C referred to below, and is not generally negligible.

If $n\phi$ is constant the eddy loss must be constant since it is proportional to $n^2\phi^2$, hence the primary component to balance eddies must be constant.

Since the air gap is large, we can assume that $\phi \propto i$ approximately where i is the magnetising current. Hence we might write $i = \frac{k}{n}$, where k is some constant.

$$\text{Also hysteresis loss} \qquad \propto \phi^{1\cdot 6} \cdot n,$$

$$\text{or} \qquad \propto n\phi \cdot \phi^{0\cdot 6},$$

so that hysteresis loss increases with increase of ϕ or decrease of n.

Considering only eddies and magnetising current, we have

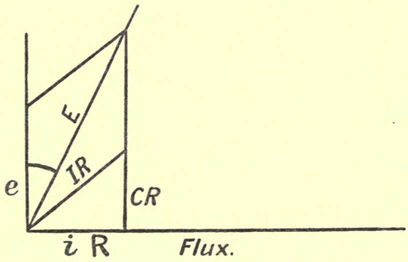


FIG. 111.

I, Total current. i, Magnetising current.
C, Load current. e, Back E.M.F.

$$I = \sqrt{C^2 + \left(\frac{k_2}{n}\right)^2},$$

$$E = \sqrt{(e + CR)^2 + \left(\frac{k_2}{n}R\right)^2}.$$

Hence, since C and E are constant by assumption, if the periodicity is raised, the E and I, the terminal volts and current, both decrease, $n\phi$ being kept constant. If E is constant $n\phi$ must increase as the periodicity is raised.

Generally also if n increases, since

$$E = e + cr,$$

c diminishes, therefore cr diminishes and E increases. Hence $n\phi_{sh}$, to which E is proportional, increases.

The phase difference between terminal volts and flux is given by

$$\text{Cot}\,\psi = \frac{i\text{R}}{e + \text{CR}},$$

ψ increases with increased periodicity ($n\phi$ constant), since i decreases, hence $90 - \psi$ decreases with increased periodicity.

Since

$$E = \sqrt{\{e + \text{R}(\text{C} + \text{hysteresis current}\}^2 + (\text{magnetising current} \times \text{R})^2},$$

we see that if $n\phi$ is content, hysteresis current and magnetising current decrease with increasing periodicity.

Also

$$\text{Cot}\,\psi = \frac{i\text{R}}{(e + \text{CR}) + (\text{hysteresis current})\text{R}},$$

both numerator and denominator decrease with increasing periodicity, and it is difficult to say whether or not tan ψ will increase or decrease, since qualities of iron are involved.

In the above theory it will be noticed the length of the eddy filaments acted upon is not referred to, although they must be involved in the action. A mathematical

examination taking into account the phase differences at different depths between different layers of eddies and their form is beyond the scope of this treatise, the problem being extremely difficult.

It appears, therefore, from the above examination that a meter may increase or decrease its speed with increasing periodicity depending upon the proportions of shunt and series, fluxes and eddies. Some examples of this are given below.

Another important matter is this. Some writers prefer to use the ordinary rotating field theory in dealing with induction meters which to us appears hardly justifiable for a meter with permanent brake magnets and a "gliding" or "sliding" field. It is, however, worth noting that if one assumes a theory of two fields rotating in opposite directions with slightly differing angular velocities, it introduces a term

$$(\phi_s^2 + \phi_{sh}^2)\omega,$$

where ω is the angular velocity of the disc, as a braking torque.

Effect of Periodicity.—In order to obtain some numerical data the following meters were tested:

Meter A.—Chamberlain and Hookham, 5 amperes, 100 volts, 50 cycles, 500 watt seconds per revolution of the disc.

The general arrangement will be understood from Figs. 112, 113.

Meter B.—Siemens type W. 10, 200 volts, 10 amperes, 50 cycles, 59·5 R.P.M. of disc with 2000 watts.

The arrangement is shown in Fig. 114.

This meter was fitted with a quadrature device shown in Fig. 114.

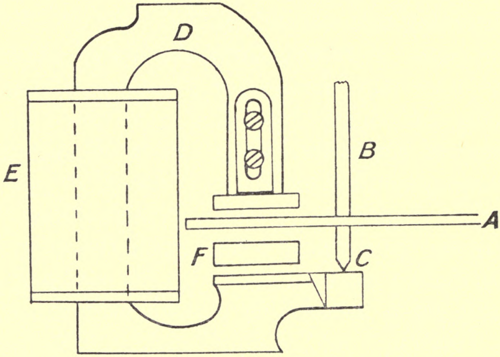


FIG. 112.—Meter A ; elevation.

A, Disc.
B, Spindle.
C, Lower bearing.
D, Iron coil.
E, Shunt coil.
F, Series coils.

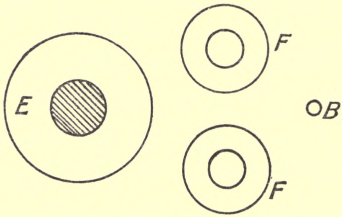
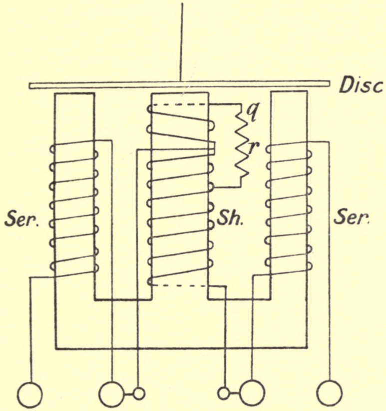


FIG. 113.—Meter A.

Diagram showing position of coils.

Meter C was a B.T.H. meter, 3 amperes, 100 volts 50 cycles.

Its arrangement is shown in Figs. 115, 116.



q, Quadrature device.
r, Resistance.

FIG. 114.—Meter B.

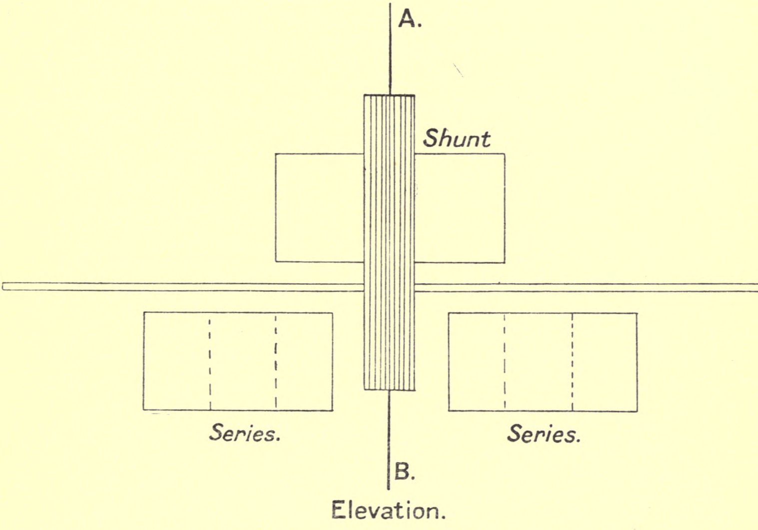


FIG. 115.—Meter C.

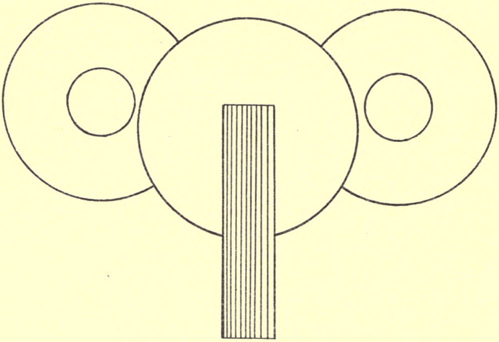


FIG. 116.—Meter C.

EFFECT OF PERIODICITY—RESULT OF TESTS

METER A. LOAD NON-INDUCTIVE

Volts.	Current.	Revolutions.	Time in Seconds.	Alternator Speed, 4 Pole.
100	300	25	40	1530
			40	1400
			40	1190
			41	1000
			44	840
			48	670

[TABLE

Meter B. Non-inductive Load. Quadrature Device cut out

E.	Current.	Revolutions.	Time.	Speed of Alternator, 8 pole.	Time of 40 Revs. (200 volts, 5·85 amperes).
			78·6	570	78·6
			70	680	70
200	5·85	40	64	805	64
			61·6	890	61·6
			60	990	60
				4 pole.	
200	9·7	...	60	885	99·4
201	9·95	40	52	980	89
200	9·90	...	42·4	1320	71·6

This meter was tested with the Quadrature Device in circuit, and the following results were obtained :

Meter B. With Quadrature Device

E.	Current.	Revolutions.	Time.	Speed of Machine, 8 pole.	Time of 40 Revolutions.
			77	590	77
			67	780	67
200	5·85	40	70·8	690	70·8
			63	915	63
			62·6	1015	62·6
				4 pole.	
200	10	...	57·2	880	97·6
200	9·95	...	50·4	1050	85·6
201	10	...	42	1350	71·6
188	9·3	40	67·2	800	100

METER C. RESULTS WITHOUT QUADRATURE DEVICE AND ONLY ONE ELEMENT EXCITED

Volts.	Current.	Revolutions.	Time in Seconds.	Speed, 4 Pole.	Remarks.
76	2·3	...	102	300	Non-inductive load.
92	2·9	...	60·4	450	
92	2·9	...	62·8	625	
92	2·9	10	72	830	
92	2·9	...	73·6	1350	
95	2·95	...	75	890	8-pole alternator.
93	2·9	...	74	1280	

METER C. QUADRATURE DEVICE IN CIRCUIT—ONE ELEMENT EXCITED

Volts.	Current.	Revolutions.	Time in Seconds.	Speed, 4 Pole.	Remarks.
92	2·9	...	60	550	Non-inductive loads.
92·5	2·9	...	68	700	
92	2·9	10	76·8	970	
92	2·9	...	60·6	410	
76	2·3	...	100	300	

[TABLE

Meter C. Effect of exciting Second Element Quadrature Device in Circuit—Second Element excited and not excited

Volts.	Current.	Time.	Speed, 8 Pole.	Revolutions.	Remarks.
90	2·8	78·4	690	...	
92	2·89	77	710	10	
92	2·8	82·2	1285	...	
90	2·8	77	690	...	Second element not excited.
92	2·89	75·2	710	10	
94	2·86	78	1285	...	

Consequently, exciting the second element produced in this case a slight decrease in speed.

The connections of this meter are given in Fig. 117.

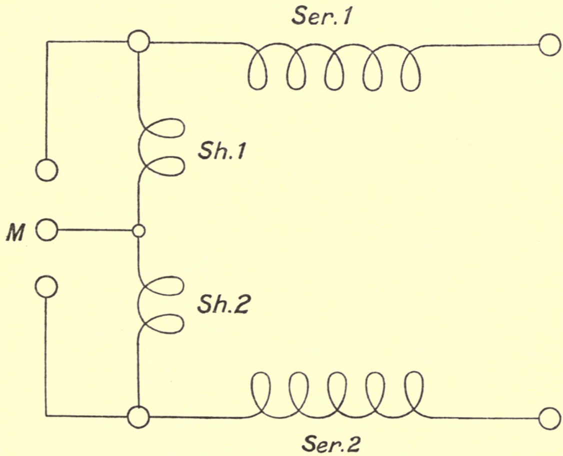


Fig. 117.—Connections of Meter C.
M, Three phase supply mains.

These tests, therefore, show that meters A and B run faster with increased periodicity, whereas C runs slower.

Data regarding currents, resistance and energy consumed in above meters.—A test was arranged as follows :

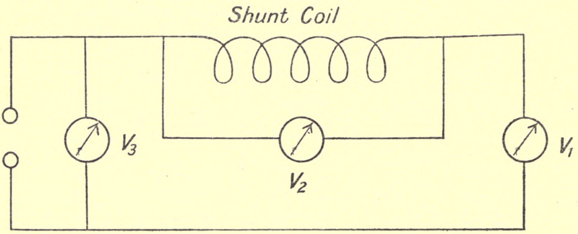


FIG. 118.

V_1 is a voltmeter of negligible reactance known to be unaffected by periodicity, V_2 is an electrostatic voltmeter. Knowing the value of V_1 the current through it could be calculated from its own reading, and by means of the 3 voltmeter method, the following results were obtained :

RESULT OF TEST

Periodicity.	V_1.	V_2.	V_3.	Remarks.
40	85	100	147	Meter C. Quadrature Device out.
45	76	100	140	
50	68	100	134	
40	29	100	124	Meter A.
45	26	100	121	
50	23	100	118·3	

Resistance of V_1 is 690 ohms.
Resistance of meter A, 514 ohms.
Resistance of meter C, 72 ohms.

RESULT OF TEST (*contd.*)

Periodicity.	Current.	Ohmic Drop.	IR.	Total Watts.	Eddy and Iron Losses.
40	·123	8·87	1·08	3·16	2·07
45	·110	7·93	·87	2·76	1·89
50	·098	7·10	·70	2·36	1·66
METER A.					
40	·042	23·7	·967	3·30	2·33
45	·037	21·2	·797	2·92	2·13
50	·033	18·8	·626	2·51	1·89

In the case of meter C, matters are complicated by the fact that there is a choking coil in series with the shunt coil proper:—

If constant volts are kept across both, as frequency increases, volts across the choking coil fall, and across the shunt coil they rise to a small extent. The following table gives results of a test:—

RESULT OF TEST

Periodicity.	Total Volts.	Choking Coil.	Shunt Coil.
34	181	109	73
51	182·5	111	73
81	181·5	111·5	71
96	181	110	70

Data regarding the shunt coil of meter C are as follows :—

Periodicity.	40.	45.	50.	Remarks.
V_2	100	100	100	
V_1	85	76	68	Resistance of V_1 is 690 ohms.
I	0·123	0·110	0·098	
IR	8·87	7·93	7·10	
I^2R	1·085	0·874	0·700	Watts by 3 voltmeter method.
Total watts	3·16	2·76	2·36	
Eddy and iron loss	2·075	1·89	1·66	
e	91	92	93	
c	0·0348	0·0300	0·0252	
$I^2 - c^2$	0·009	0·01120	0·00906	
i	0·118	0·106	0·095	
in	0·473	0·477	0·475	Constant to within 1%.

Temperature Error in Induction Meter.—These errors are generally referred to as very great, owing to the change of specific resistance in the rotating disc.

Now it seems clear that if the eddies in the disc due to the series and shunt magnetic fluxes encounter resistance, this resistance enters into the denominator of the expression for driving torque. Call it R_0, then at temperature $\theta°$ it will be $R = R_0(1 + a\theta°)$ provided ohmic resistance only is referred to. In the same way the braking torque must have some resistance R'_0 in its denominator, and at temperature $\theta°$ it will be $R = R'_0(1 + a\theta°)$. Hence for a rise in temperature $\theta°$,

$$\frac{kn}{R'_0(1 + a\theta)} = \frac{T}{R_0(1 + a\theta)},$$

or the temperature correction cancels out. If it affects the numerator owing to the length of eddy current paths,

it must be so to a very small extent, and the difference between this effect in braking and driving torques only can be active.

The Chamberlain and Hookham meter referred to above was run at constant load at a room temperature of 21° C., and the revolutions of the disc were 50 in 84 seconds. On warming the meter by placing an incandescent lamp inside its case till a thermometer placed on the disc read 40° C., and by shunt resistance, the temperature rise of the shunt coil (average) was 30° C., yet the reading of speed was 50 revolutions in 85 seconds. Consequently, any error due to ordinary temperature rises is quite a small quantity. We have already dealt with the effect of variations in the resistance of the shunt coil circuit, and if temperature affects them, the corresponding effect on eddies induced by flux change can be roughly seen.

Effect on Variation of cos λ.—Exhaustive experiments carried out by means of a phase-changing device show that the expression :

$$\frac{\text{Reading of meter in revolutions per second}}{\cos(\lambda + \delta)}$$

for a given value of volt amperes is constant within wide limits.

It will be noticed as $(\lambda + \delta)$ approaches 90° the meter will stop and will creep backward or forward according as $\lambda + \delta >< \frac{\pi}{2}$. Where $\delta = 90 - (a - \gamma)$, see footnote, p. 267. A phase shift of three-quarters of a degree was sufficient to make this effect apparent in the case of the Chamberlain and Hookham meter referred to above, viz. Meter A.

Quadrature Devices for Induction Meter

In order to keep the flux due to the shunt coil 90° out of phase with the terminal volts on the meter when it is of the induction type, several quadrature devices are used.

The most usual is merely a few turns of winding on the limb of the shunt magnet, forming a closed secondary of a leaky transformer. By adjusting the resistance of this closed circuit, it is possible to obtain approximate compensation.

Another method used is that shown in the figure :

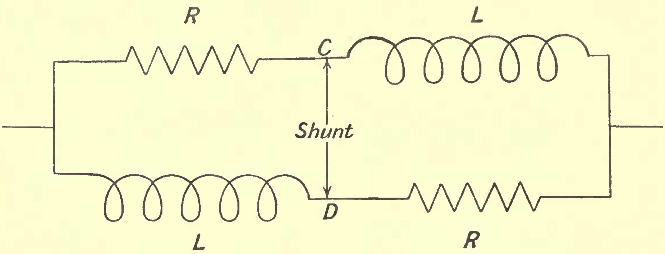


Fig. 119.—Quadrature device.
R, R, Resistances. L, L, Choking coils.

The shunt magnet coil is then connected between the points CD. Using sine curve theory it is seen that whatever values R, L may have, the locus of the apex of the vectorial triangle lies always in a circle. Hence by adjusting R, L, we can obtain any desired phase angle between the volts and flux.

It must not be forgotten, however, that hysteresis and eddies introduce complications into the simple theory.

Take, for instance, the case of a compensating device, such as :

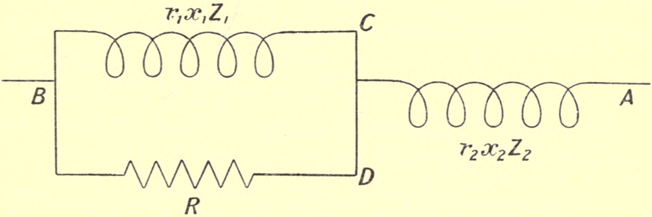


FIG. 120.—Quadrature device.

where R is a non-inductive resistance, $r_1 x_1$ is the meter shunt coil, $r_2 x_2$ is the choking coil, Z_1, Z_2 the impedances. The required condition to be satisfied is that the current in BC lags 90° behind the voltage across AB.

Let the current in BC be

$$i = \mathrm{I} \sin pt,$$

volts across BC

$$= e_1 = \mathrm{E}_1 \sin (pt + \phi_1),$$

or

$$e_1 = \mathrm{IZ}_1 \sin (pt + \phi_1).$$

The current in BD is

$$= \frac{\mathrm{IZ}_1 \sin (pt + \phi_1)}{\mathrm{R}},$$

current in AD

$$= \mathrm{I} \sin pt + \frac{\mathrm{IZ}_1}{\mathrm{R}} \sin (pt + \phi_1),$$

volts across AD $= e_2$

$$= \mathrm{IZ}_2 \left\{ \sin (pt + \phi_2) + \frac{\mathrm{Z}_1}{\mathrm{R}} \left(\sin pt \,.\, \cos(\phi_1 + \phi_2) + \cos pt \,.\, \sin(\phi_1 + \phi_2) \right) \right\}$$

or volts across $\mathrm{AB} = e_1 + e_2$

$$
\begin{aligned}
&= \mathrm{IZ}_1 \sin(pt + \phi_1) \\
&\quad + \mathrm{IZ}_2 \left\{ \sin(pt + \phi_2) + \frac{\mathrm{Z}_1}{\mathrm{R}} \left(\sin pt \cos(\phi_1 + \phi_2) \right.\right. \\
&\qquad\qquad \left.\left. + \cos pt \sin(\phi_1 + \phi_2) \right) \right\} \\
&= \mathrm{I} \left\{ \sin pt \left(\mathrm{Z}_1 \cos \phi_1 + \mathrm{Z}_2 \cos \phi_2 + \frac{\mathrm{Z}_1 \mathrm{Z}_2}{\mathrm{R}} \cos(\phi_1 + \phi_2) \right) \right. \\
&\qquad \left. + \cos pt \left(\mathrm{Z}_1 \sin \phi_1 + \mathrm{Z}_2 \sin \phi_2 + \frac{\mathrm{Z}_1 \mathrm{Z}_2}{\mathrm{R}} \sin(\phi_1 + \phi_2) \right) \right\}
\end{aligned}
$$

The necessary condition is that

$$
\phi_3 = \tan^{-1} \frac{\mathrm{Z}_1 \sin \phi_1 + \mathrm{Z}_2 \sin \phi_2 + \dfrac{\mathrm{Z}_1 \mathrm{Z}_2}{\mathrm{R}} \sin(\phi_1 + \phi_2)}{\mathrm{Z}_1 \cos \phi_1 + \mathrm{Z}_2 \cos \phi_2 + \dfrac{\mathrm{Z}_1 \mathrm{Z}_2}{\mathrm{R}} \cos(\phi_1 + \phi_2)} = \frac{\pi}{2}
$$

or that

$$
\mathrm{Z}_1 \cos \phi_1 + \mathrm{Z}_2 \cos \phi_2 + \frac{\mathrm{Z}_1 \mathrm{Z}_2}{\mathrm{R}} \cos(\phi_1 + \phi_2) = 0.
$$

This finally reduces to the condition

$$
x_1 x_2 = \mathrm{R}(r_1 + r_2) + r_1 r_2,
$$

which can only be true for one periodicity.

For low periodicities we see that $\tan \phi_3$ is +ve and $\phi_3 < \dfrac{\pi}{2}$.

For high periodicities we see that $\tan \phi_3$ is −ve, and $\phi_3 > \dfrac{\pi}{2}$.

The following experiment on a highly inductive load was made :

An alternator was connected with a choking coil in

series with the current coil of a meter. The machine was given its maximum excitation, and the speed varied over the whole available range. A wattmeter in the circuit gave a low reading, and this was read as accurately as possible.

It will be seen that in all cases the power factor was low. The speed of rotation of the disc was low, so that the accuracy of reading was not very great.

The following results were obtained :

RESULTS OF EXPERIMENT

Volts.	Current.	Watts approx.	Power Factor.	Quadrature Coil. Time of One Revolution. In.	Out.	Speed, 4 Pole.
38	5·1	25	0·129	+ 21	+ 18·5	400
83	5·5	25	·055	+ 61·2	+ 20	950
119	5·6	30	·045	– Just moves	+ 22	1200
143	5·6	30	·038	– 27	+ 24	1320

In all the cases where the +ve sign is used, the meter runs backward; where –ve is used it runs forward.

Watts.—The resistance of the choking coil was about 1 ohm, hence the copper loss at 5 amperes about 25 watts. The iron loss at 50 cycles—120 volts—is less than 10 watts, and is very difficult to estimate.

It will be noticed that as the speed increased with the quadrature coil cut out, the speed of the meter decreased, but is always in the reverse direction ; with the coil in, a change in direction is produced.

Consequently, with the coil out, the quantity

$$\frac{\pi}{2} - \left\{ \begin{array}{l} \text{phase difference between applied volts} \\ \quad \text{and shunt coil flux} \end{array} \right\}$$

is always > angle of lag in the series circuit, whereas, with it in, it starts by being greater, but is ultimately less.

The Aron Meter

This is probably the most interesting of all the meters, besides being the most accurate. It is based on the fact that for small oscillations of a simple pendulum the time of swing is given by

$$t = 2\pi\sqrt{\frac{l}{g}},$$

where l is the length of the pendulum and g is the gravitational constant. If one imagines that g can be altered in any way, the periodic time of swing will also be altered.

Suppose, then, a pendulum bob to be acted upon by gravity and also by a field of force due to solenoid proportional to C (see Fig. 121), then the time of swing

$$t = 2\pi\sqrt{\frac{l}{g + k\mathrm{C}}}$$

where k is some constant, and since

$$t = \frac{1}{n_1}$$

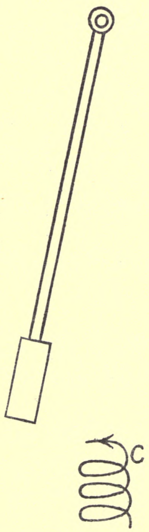


Fig. 121.—Diagram of Aron Meter.

$$n_1 = \frac{\sqrt{g + k\mathrm{C}}}{2\pi\sqrt{l}} = \frac{g^{\frac{1}{2}} + \frac{1}{2}g^{-\frac{1}{2}}k\mathrm{C} +, \text{ etc.}}{\text{constant}}.$$

If n_0 be the time of swing due to gravity only, then

$$n_0 = \frac{\sqrt{g}}{\text{constant}},$$

$$\therefore \quad n_0 - n_1 = \text{AC},$$

where A is some constant and C is the current strength. This assumes that the other terms in expansion are negligible—in reality

$$n_0 - n_1 = \text{AC} + \text{BC}^2,$$

where B is some constant. Consequently, the equation to $n_0 - n_1$ would be a parabola, and only approach a straight line if C was very small.

The original meter was based upon the above principle, the connecting train being so arranged that it only registered the difference between the numbers of oscillations when current was flowing.

If now two pendulums are so arranged as to be acted upon in opposite senses by the magnetising forces, as shown in Fig. 122, where S*h* are the coils on the pendulums actuated by volts, and S*er* the main series coils, we see that the attraction or repulsion between them is proportional to I*i*, or IE, viz. watts. Call this W. We have then

$$n_1 = \frac{\sqrt{g + k\text{W}}}{c}, \quad n_2 = \frac{\sqrt{g - k\text{W}}}{c}.$$

Expanding by Binomial Theorem,

$$n_1 - n_2 = \frac{1}{c}\Big(g^{\frac{1}{2}} + \tfrac{1}{2}k\text{W}g^{-\frac{1}{2}} - \frac{1}{2}\cdot\frac{1}{2}g^{-3/2}\frac{k^2\text{W}^2}{2} +, \text{etc.}$$

$$-g^{\frac{1}{2}} + \tfrac{1}{2}k\text{W}g^{-\frac{1}{2}} + \frac{1}{2}\cdot\frac{1}{2}g^{-3/2}\frac{k^2\text{W}^2}{2} +, \text{etc.}\Big)$$

$$\therefore \quad n_1 - n_2 = A \cdot W,$$

since the term involving the square now cancels out, and A is some constant.

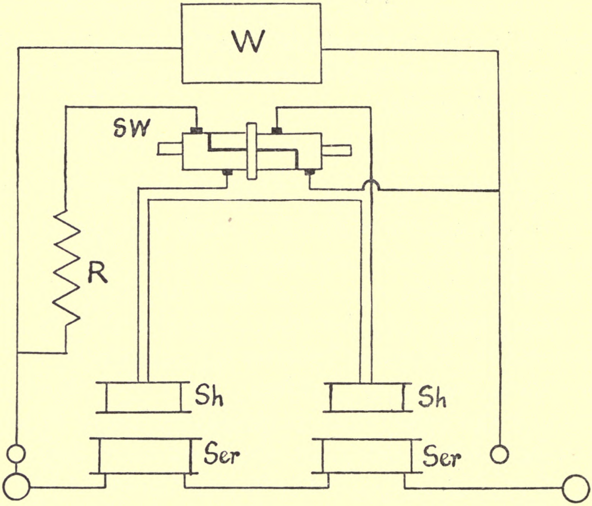


FIG. 122.—Aron Meter.

SW, Reversing Switch. W, Winding Gear.

Consequently the difference in the readings at any instant represents

$$(n_1 - n_2) \times t = Wt, \text{ or watt hours.}$$

The Aron Meter possesses many advantages, amongst which are the following. It is a true dynamometer watt-hour meter; it is, except perhaps for three phase work, unaffected by stray magnetic fields; and it is unaffected by wave form, or frequency. It is suitable for

both continuous and alternating circuits, and, except in the continuous current watt-hour meter, contains no iron in its construction.

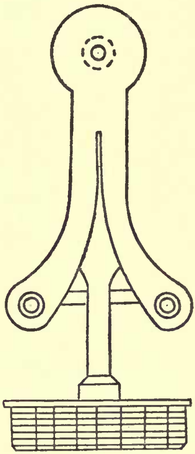


FIG. 123.—Pendulum of Aron Meter.

We have both direct and alternating meters. The only difference is that the laminations of the magnet of the winding gear are thinner in the A.C. than the D.C. meter.

Owing to the resistance wire in series with the shunt coils having an almost negligible temperature coefficient and the series coils carrying the current passing through the circuit, the temperature error is practically negligible.

Generally A.C. above 300 amps., use current transformer. D.C., no shunt is used up to 1000 amps. Shunted 800 to 5000 amps.

Again, its accuracy is the same at all loads, which is an important matter.

The chief disadvantages of this type are difficulty of testing. It is not easy to say after a brief examination whether the meter is reading correctly or not. The location of faults is a difficult matter. Other troubles referred to are of the purely mechanical type, owing to the number of contacts necessary, and the complex clockwork mechanism.

Reversing Gear.—Periodically the connections to the shunt circuit are reversed, thereby eliminating any error due to unequal isochronism between the pendulums.

Let l_1, l_2 be the equivalent lengths of the pendulums, assume the watts to remain constant over two successive intervals of about ten minutes, then

$$n_1 = \frac{\sqrt{g+kw}}{2\pi\sqrt{l_1}}, \quad n_2 = \frac{\sqrt{g-kw}}{2\pi\sqrt{l_2}},$$

may be written in the form

$$\frac{1}{2\pi}, \sqrt{\frac{g}{l_1}}\sqrt{1+\frac{kw}{g}}.$$

If N_1, N_2 are the oscillations per second after reversal, then the quantity recorded is $\{(n_1 - n_2) + (N_2 - N_1)\}t$, and it is easily shown by expanding, this is

$$\frac{t}{2\pi}\sqrt{g}\left\{\frac{1}{\sqrt{l_1}} + \frac{1}{\sqrt{l_2}}\right\}kw + \text{terms of third order.}$$

The reversing gear is generally used, and no attempt to make the pendulums isochronous at no load is made.

Motor Meter

Starting and Stopping.—In meters of the Chamberlain and Hookham type, or improved Ferranti, where the torque accelerating is due to current and resisting torque, proportional to the speed, we have the following :

$$i = \frac{E - e}{r}.$$

Accelerating torque $= k_1 i =$ constant.

The resisting torque $= k_2 n$.

Writing M for moment of inertia,

$$\frac{d^2\theta}{dt^2}=\frac{k_1 i-k_2 n}{\mathrm{M}}.$$

And since

$$\frac{d^2\theta}{dt^2}=\frac{dn}{dt},$$

$$\mathrm{M}\frac{dn}{dt}=k_1 i-k_2 n,$$

or

$$\mathrm{M}\frac{dn}{dt}+k_2 n=k_1 i.$$

This being of the form

$$\frac{dx}{dt}+\mathrm{P}x=\mathrm{Q},$$

we have for its solution

$$n=\epsilon^{-\frac{k_2 t}{\mathrm{M}}}\left\{\frac{k_1}{\mathrm{M}}i\epsilon^{\frac{k_2 t}{\mathrm{M}}}+\text{const.}\right\}$$

or

$$n=\frac{k_1}{\mathrm{M}}i+\epsilon^{-\frac{k_2 t}{\mathrm{M}}}\times\text{const.}$$

Now $\frac{k_1}{\mathrm{M}}i$ corresponds to the final speed n_0 when acceleration ceases,

$$n=n_0+\epsilon^{-\frac{k_2 t}{\mathrm{M}}}\times\text{const.}$$

Now if when $t=0$, $n=0$, then const. $=-n_0$,

$$\therefore\quad n=n_0\left\{1-\epsilon^{-\frac{k_2 t}{\mathrm{M}}}\right\}.$$

Theoretically, therefore, the motor takes an infinite

time to reach the speed n_0, $n = n_0$ being the asymptote to the speed curve.

Again, on stopping the motor supposed to be running at some speed n_0, we have

$$\frac{dn}{dt} = -\frac{k_2 n}{M},$$

$$\int_{n_0}^{n} \frac{dn}{n} = -\frac{k_2 t}{M},$$

or
$$n = n_0 \epsilon^{-\frac{k_2 t}{M}}.$$

If
$$t = \frac{M}{k_2},$$

we have
$$n = n_0 \epsilon^{-1},$$

or
$$n = \frac{1}{\epsilon} n_0,$$

and if the speed is rising at the time

$$t = \frac{M}{k_2},$$

$$n = n_0(1 - \epsilon^{-1}),$$

or
$$n = n_0 \frac{\epsilon - 1}{\epsilon}.$$

Now, in such a case, if the area integrated is that shown in Fig. 124, we see that the correct area is $n_0 \tau$, *i.e.* the area of a, b, c, d. The meter, however, reads too little at starting and too much on stopping. The amount registered by the meter is

$$n_0\int_0^{\tau}\left(1-\epsilon^{-\frac{k_2t}{\mathrm{M}}}\right)dt+n_0\int_0^{\tau}\epsilon^{-\frac{k_2t}{\mathrm{M}}}dt,$$

or
$$n_0\int_0^{\tau}dt=n_0\tau.$$

Hence a meter with torque accelerating aI, or EI, and retarding torque proportional to n, will read correctly on fluctuating loads.

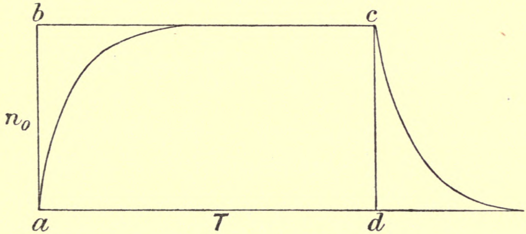


FIG. 124.—Starting and stopping.

Since
$$\frac{d^2\theta}{dt^2}=\frac{\text{moment of forces}}{\text{moment of inertia}},$$

putting M for moment of inertia, we have for a meter of the Ferranti type, since

$$\frac{dn}{dt}=\frac{d^2\theta}{dt^2},$$

$$\mathrm{M}\frac{dn}{dt}=k_1\mathrm{I}^2-k_2n^2.$$

If C has some value C_0, and is suddenly switched off when $n=n_0$, we see that

$$\frac{dn}{dt}=-\frac{k_2}{\mathrm{M}}n^2,$$

$$\int_{n_0}^{n} \frac{dn}{n^2} = -\frac{k_2}{\mathrm{M}}t,$$

or
$$\frac{1}{n_0} - \frac{1}{n} = -\frac{k_2 t}{\mathrm{M}}.$$

Again in accelerating we have

$$\frac{dn}{dt} = \frac{k_1 \mathrm{I}^2 - k_2 n^2}{\mathrm{M}},$$

and writing A and B for $\frac{k_1}{\mathrm{M}}$ and $\frac{k_2}{\mathrm{M}}$ respectively,

$$\int_0^{n_0} \frac{dn}{\mathrm{AI}^2 + \mathrm{B}n^2} = \int_0^t dt,$$

$$\mathrm{B} \log_\epsilon \frac{\mathrm{AI}^2 - \mathrm{B}n^2}{\mathrm{AI}^2} = t,$$

$$\frac{\mathrm{AI}^2 - \mathrm{B}n^2}{\mathrm{AI}^2} = \epsilon^{-t/\mathrm{B}}.$$

But when the meter ceases to accelerate,

$$\mathrm{AI}^2 = \mathrm{B}n_0^2,$$

so substituting we have

$$\mathrm{B}n^2 = \mathrm{B}n_0^2(1 - \epsilon^{-t/\mathrm{B}}),$$

or
$$n^2 = n_0^2(1 - \epsilon^{-t/\mathrm{B}}).$$

The counting train therefore indicates

$$n = n_0 \sqrt{(1 - \epsilon^{-t/\mathrm{B}})},$$

or the R.M.S. values of revolutions.

Now we see that the revolutions integrated in accelerating will be

$$\int ndt = n_0 \int_0^t (\sqrt{1-\epsilon^{-t/\mathrm{B}}})dt,$$

and on retarding

$$\int ndt = \int \frac{dt}{\frac{1}{n_0} + \mathrm{B}t},$$

$$\therefore \quad \int ndt = \frac{1}{\mathrm{B}} \log_\epsilon \left(\frac{1}{n_0} + \mathrm{B}t \right) + \text{const.}$$

It is obvious from these expressions that the area lost on accelerating is different from the area gained on retarding, and consequently there will be an error involved on fluctuating loads with this type of meter.

Errors in Meters caused by Short Circuits, etc.

In a paper (*Jour. I.E.E.* vol. li. p. 223), "Some Recent Improvements in Continuous Current Meters," by W. E. Cooke, this question is discussed.

It appears that in Ferranti meters of the old type, the meter actually improves for a time after being short circuited, but gradually returns to its old régime.

One hundred per cent excess current had apparently no effect on Chamberlain and Hookham meters, and less than 2 per cent on Ferranti meters.

The temperature error of mercury motor meters is given as 0·2 per cent per degree Fahrenheit.

Watt-hour meters of the Elihu Thomson type appear to have from the curves given only about ½ per cent error at full load, and about the same at ¼ load, the

error increasing below $\frac{1}{10}$ load. In the old type meter, roughening of the commutator by grit and dust slowed them down, after eighteen months' use at Cape Town.

The two rate meters, both Elihu Thomson and Aron type, worked exceedingly well.

A factor causing meters to work badly was their being fixed to walls subjected to vibration, which may be of importance in streets through which heavy traffic passes. In the Reason Meter a platinum gauze fence is used to prevent mercury being spilt. This acts very well—hard banging on the bench to which meter is fixed causing none to come over.

For a description of the Solar Hydrogen Meter, the reader should consult *Electrician*, 22nd May, 1914, p. 265, article by H. S. Hatfield, B.Sc.

Accuracy of Instruments

Regarding this question the reader might consult the *Standard Specification of the Engineering Standards Committee.* This was published in 1907, and is now somewhat out of date.

A paper by Mr. S. H. Holden (*Inst. of E.E.*, Birmingham Section), and also *Electrician* of November 14, 1913, and for discussion same paper of January 23, 1914, might also be consulted. The following suggestions were put forward in the paper referred to for consideration :

Starting Currents—Supply Meters.—That first-class single phase meters shall start with $\frac{1}{2}$ of 1 per cent of full load current when power factor = 1. For continuous currents 0·05 ampere if under 10 amperes capacity, and $\frac{1}{2}$ of 1 per cent full load if over 10 amperes capacity.

Errors.—$2\frac{1}{2}$ per cent not to be exceeded between full and $\frac{1}{10}$ load. At any range below $\frac{I}{N}$ load the error not to exceed $\frac{N}{4}$. This is the wording of the specification, and as it is obviously quite empirical, it is open to criticism. At present alternating meters can easily be made to read to within 2 per cent from $\frac{1}{20}$ load.

Variation of Pressure.—Generally 10 per cent variation should not cause more than 1 per cent error in continuous current meters. For alternating current 5 per cent should not cause 1 per cent error.

Power Factor.—Variation of cos ϕ from 1 to 0·5 should cause not more than 2 per cent error.

Wave Form.—This is not dealt with in the *Standard Specification.* Mr. Holden suggests that a variation of 10 per cent from a true sine curve at any point should not cause more than a 1 per cent error. This obviously leaves the matter open for further discussion, more especially as sine curves are not met with in practical work.

CHAPTER IX

MAGNETIC TESTING INSTRUMENTS

The Koepsel Permeameter

Fig. 125 shows this interesting instrument. It consists essentially of a magnetic circuit formed by the specimen air gaps and pole pieces PP as shown. The air gap is only 1·5 m/mms., and the depth is 8 cms., so that magnetic resistance is reduced. A magnetising solenoid is wound on the specimen, about 12 cms. long, and a coil moves in the air gap.

The method of using it is as follows :

Determine the area of the specimen, calculate current through moving coil (C) from the relation

$$\text{Current} = \frac{\text{constant}}{\text{area}}, \text{ usually } \frac{0{\cdot}005}{\text{area}},$$

then energise the moving coil, and note position of pointer. If the pointer is not at zero it is turned until that condition is satisfied.

Stray lines will then be perpendicular to the line joining the poles. Insert the specimen, using split bushes of soft iron to make it fit holes, if necessary, and clamp it. Demagnetise it by reversal and when the instrument reads zero, the specimen is demagnetised. Increase the magnetising current by steps and read B

and magnetising current. The magnetising current is given by

$$H=\frac{4\pi ni}{10l}=ki$$

where $k=100$ usually.

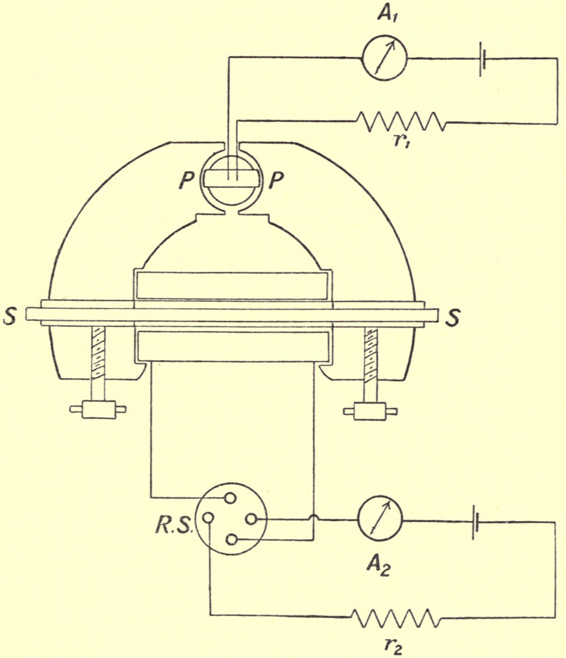


FIG. 125.—The Koepsel Permeameter.

S, Specimen. P, Pole pieces. A_1, A_2, Ammeters.
R.S., Reversing switch. r_1, r_2, Regulating resistances.

Taking $H=\frac{4\pi ni}{10l}$, since l is usually about 12 cms., n is about 1000. Finally H must be corrected for shearing, if necessary.

As the instrument measures B directly, it is interesting.

By means of it, the usual tests can be carried out, such as hysteresis loops, B—H curves, etc.

In order to compensate for the magnetising effect of the solenoid carrying specimen acting on the yokes or pole pieces, these are wound with a magnetising coil in opposition to that on the solenoid and in series with it.

The following tables give the results of tests of steel rod and cast iron :

Test of Steel Rod
Area, ·284 sq. cms.

B.	H.	H.
Kilolines per sq. cm.		
0	27·2	– 27·2
1	28·6	– 26·1
2	30	– 25·5
3	31·8	– 23
4	33·3	– 21·3
5	35	– 19·5
6	37	– 17·2
7	39	– 14·4
8	41·9	– 10·6
9	45	– 5·3
10	49·4	1·5
11	55	9·3
12	62	19·5
13	71·5	32
14	84·5	49
15	106	74
16	145	111
17	210	176
18	350	350

STEEL SHEARING

B.	H.	H.
0	1	– 1·0
1	1·2	– 1·0
2	1·5	– ·8
3	1·9	– ·5
4	2·0	– ·1
5	2·2	– ·05
6	2·8	·05
7	3·1	·2
8	3·5	·8
9	4·0	1·0
10	4·8	1·2
11	5·5	2·0
12	6·9	2·4
13	8·5	3·0
14	11·0	5·4
15	15·0	8·5

TEST OF CAST IRON. Area, 0·283 sq. cms.

B.	H.	H.
0	11	– 11
1	13	– 9·7
2	15·4	– 8
3	18·8	– 6
4	23·5	– 3
5	30·8	3
6	41·0	15
7	58	33
8	82	60
9	118	101
10	190	170
10·1	202	202

The results giving shearing corrections were obtained by comparing ballistic results from a solid ring of the same material as that tested by the instrument.

The following results illustrate the use of the instrument for testing residual magnetism.

Max. B.	Residual B.
16·2	7·0
14	6·5
10	5·5
7	4·5
5	3·6
2·5	2·0

Similarly, the usual effects of hysteresis when working between different limits may be studied.

It is unnecessary to refer to Ewing's Permeability Bridge, as it is already described in *Magnetic Induction in Iron and Other Metals.*

The Du Bois Magnetic Balance is an important instrument, and since it depends for its action on gap induction, it is of interest as an instrument. It will be noticed that the yoke is suspended eccentrically, and brought into equilibrium by means of weights moving along the scale.

The specimens have to be prepared with rounded ends as shown in the figure, and to convert the **B—H** readings obtained for shear due to the air gap, a curve or correcting factor must be used. The test bars are about 25·5 cms. long and 0·798 cm. diameter or 0·5 cm. area.

If W be the weight of the movable portion of yoke, its C.G. is at a distance k, say, from the fulcrum, or knife

edge. Let a be distance of the line of action of the force at one gap from knife edge, b the distance of the other, l distance of movable weight w.

If A are areas of the gaps, then for equilibrium

$$\frac{B^2Aa}{8\pi}+wl=Wk+\frac{B^2Ab}{8\pi},$$

$$\frac{B^2A}{8\pi}(a-b)=-wl+Wk.$$

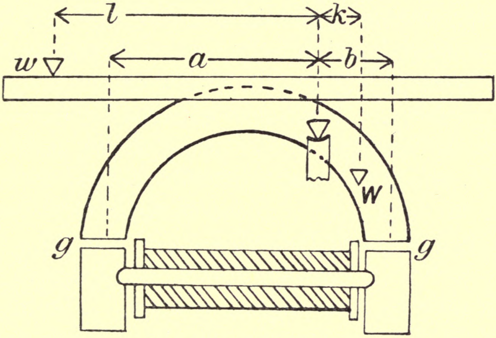


FIG. 126.—Du Bois Magnetic Balance.

If w is originally placed at a zero to neutralise Wk, then $-wl+Wk=wl'$, or

$$\frac{B^2A}{8\pi}(a-b)=wl',$$

$$B\propto\sqrt{l'}.$$

Hence l' can be marked off directly in values of B for the iron rod when the areas of gap and rod are known.

The necessity of correcting the curve for H owing to the gaps seems to the authors a very serious disadvantage.

The magnetising coil is wound so that H is numerically

equal to the strength of current in centiamperes. H can be raised to about 150 C.G.S.

In the above the pull was taken as $\frac{B^2A}{8\pi}$. Bosanquet has shown that for a narrow air gap the actual pull is about $\frac{1}{2}\frac{B^2A}{8\pi}$ dynes per square centimetre, so the proportion appears to hold.

For practical purposes, magnetic testing is of two kinds, viz. testing to obtain a knowledge of B and H for the material and purely hysteresis testing, or the measurement of magnetic induction in the air gaps of, say, a dynamo, in order to determine the flux distribution.

B *and* H *Testing*.—For accurate testing of B and H generally, either the magnetometer method or ballistic method may be used. These methods are discussed very fully in *Magnetic Induction in Iron and Other Metals*, by Professor J. A. Ewing, F.R.S., so that only a few remarks may be made regarding them. The magnetometer method has in recent years almost been given up, and the ballistic method used instead. By either of these methods a typical B and H curve or hysteresis loop may be obtained.

In the former method the thin rod of the material is placed about the centre of the magnetising solenoid, and although all error due to the demagnetising influence of the ends cannot be eliminated, yet it can be reduced to negligible dimensions, so that H is accurately known. This method enables **I** to be found in terms of the earth's magnetic intensity. Probably the fact that the earth's direction force has to be accurately known has led to the abandonment of this method for absolute work, but for rapidly comparing the **I** and H values of a specimen

with those of a standard, the method has many advantages in a place free from disturbing influences. In the opinion of the authors, this method is most valuable for comparative tests, and should be resuscitated.*

The ballistic method suffers from considerable complications. In the first place, to secure endlessness, the material is generally made in the form of a ring, or toroid, of stampings of the material to be tested, which requires correction for distribution of H unless it is of narrow rectangular section. Again the galvanometer has to be accurately calibrated either by discharging a known quantity of electricity through it or by means of comparison with a standard solenoid or by measurement of current and direct deflection. A result given in a paper by Messrs. Hadfield & Hopkinson (*Jour. Inst. E.E.* vol. xlvi. p. 270) is as follows:

Condenser . .	$3{\cdot}67 \times 10^{-7}$	coulombs	per	division.
Standard fields .	$3{\cdot}735 \times 10^{-7}$	,,	,,	,,
	$3{\cdot}745 \times 10^{-7}$	,,	,,	,,
Steady deflection	$3{\cdot}76 \times 10^{-7}$	,,	,,	,,

Period of galvanometer, 15·40 seconds.

On another day when the period of the galvanometer was 15·27 seconds, the results were:

Steady deflection $3{\cdot}715 \times 10^{-7}$.
Condenser . . $3{\cdot}715 \times 10^{-7}$.

The most accurate value of the constant is probably given by comparison with a standard field, consisting of a long solenoid, with a suitable search coil. We see, however, from the above figures there is generally a

* See also "A Direct Method of measuring Magnetic Susceptibility and an Instrument for this Purpose," by W. H. F. Murdoch, *Electrician*, Sept. 19, 1913. Paper read at Section A, British Association.

difference in the constant of about 1 per cent. Of course the accuracy of the final result depends on the calibration of the galvanometer, and it is interesting to notice (*Absolute Measurements in Electricity and Magnetism*, Gray, p. 34) that determinations of "**H**" at Glasgow University during periods in May, June, and August only showed differences in the mean values of 4 in 1520 or less than $\frac{1}{3}$ of 1 per cent, so that if the deflections in the magnetometer method could be observed with the same accuracy as the ballistic ones, the advantage is with the magnetometer method. Another objection urged against the magnetometer method is "the indeterminate position" of the poles.

There are several points in magnetic testing to be carefully borne in mind. A method may be perfectly suitable for low permeability materials, but unsuitable for high permeability. Many methods are altogether useless for inductions over 30,000 lines per square centimetre.

For instance, let $\mathbf{B} = \mu \mathbf{H}$

where $$\mathbf{H} = \frac{4\pi n \mathrm{C}}{10 l},$$

the usual expression for the magnetic intensity at the centre of a solenoid, then

$$\mathbf{B} \propto \mu \mathrm{C}.$$

Now the permeability is by no means constant and, generally speaking, for ordinary wrought-iron 20,000 C.G.S. lines is about the limit of possible magnetisation by means of coils. All methods using a coil to magnetise the iron suffer from this limitation. The limit is due to the fact that current can only be passed through the

wire till it attains its maximum permissible temperature, and also due to the fact that μ is diminishing rapidly.

This is unfortunate, since the most important matter to designers is a knowledge of **B** and **H** in sheets of iron at inductions of 30,000 or so. So far this difficulty has not been met. The Isthmus Method certainly can be used to test a small piece of material, and that is the only method we have available, or a modification of it. The **H** produced by a coil is relatively insignificant compared with that due to the free intensity of a pole face.

Amongst permeameters there is the ingenious plug permeameter of Dr. C. V. Drysdale. This is used for testing thick plates or masses of iron. A hole is drilled in the iron leaving a central pin, as shown below (Fig. 127),

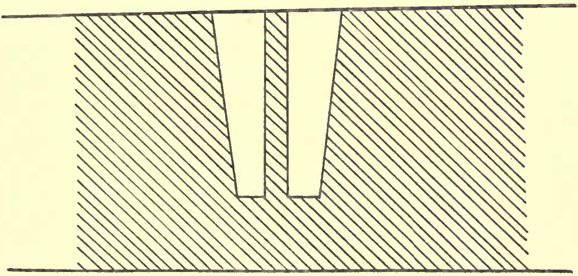


FIG. 127.—Drysdale Permeameter.

and a plug with a primary and secondary coil wound in it is inserted over the pin. The current in the primary magnetises the specimen, and the search coil can be connected to a ballistic galvanometer in the usual way.

The difficulty seems to arise in eliminating the resistance of the joints in the material, and further particulars of the errors will be found in *Proceedings of Phys. Society of London*, 1908.

For testing the permeability of low permeability materials, one of the authors devised a friction permeameter (*vide Jour. I.E.E.* vol. xl. p. 137). It consisted in one form of a bar and yoke arrangement as shown :

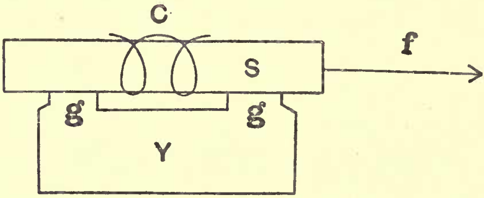


FIG. 128.—Murdoch Friction Permeameter.

S is the specimen, Y the yoke. Previous to magnetisation by the coil C the specimen may be pulled by a force f, causing it to slide slowly on the faced gaps gg. This friction pull can be measured. Then the specimen is magnetised and the process repeated.

Let W = weight of specimen,
γ = friction coefficient,
B = magnetic induction,
A = area of gaps,
f_0 = friction pull,

Then
$$f_0 = W\gamma,$$

and
$$f - f_0 = \frac{B^2 A}{8\pi}\gamma,$$

$$\therefore \quad B = \sqrt{\frac{8\pi W}{A}\,\frac{(f - f_0)}{f_0}}.$$

H can be found from the usual magnetic circuit equation. The instrument answered its purpose excellently, viz. for testing large blocks of cast-iron. Other instruments

for comparative testing were exhibited at the British Association, 1911, the principle being the same, but the top was revolved against action of a spring. In this case,

$$B = k\sqrt{\theta}.$$

Here θ is the angle measured from zero, this zero being adjusted to eliminate friction altogether.

In these instruments the magnetic circuit remained unbroken and the current was not reversed during a test. Consequently H was not applied impulsively as in ordinary ballistic work.

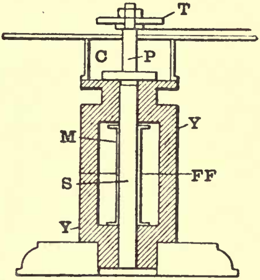


FIG. 129.—Murdoch Friction Permeameter.

S, Specimen.
M, Coil.
P, Brass pin.
FF, Gap faces.
C, Spring control.
T, Torsion head.
Spring omitted for clearness.

Several interesting methods of testing are given by

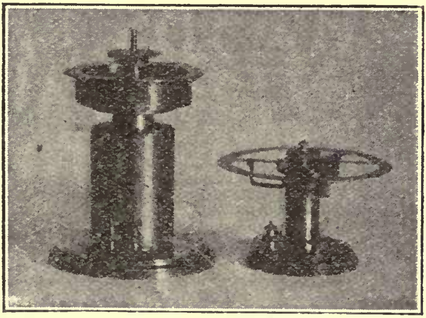


FIG. 130.—Murdoch Friction Permeameter.

A. Campbell in (*Jour. I.E.E.* vol. xxxvi. p. 221), "The

Testing of Cast-Iron and other Materials by the Ewing Permeability Bridge."

An interesting paper on "The Method of Constant Rate of Change of Flux as a Standard for determining Magnetisation Curves of Iron" (J. T. Morris and T. H. Langford, B.Sc., *Physical Society of London*) should be referred to. In this method the principle is as follows: An iron ring is wound with a primary and secondary. The current in the primary is varied in such a manner that E, the E.M.F. of the secondary, remains constant. The experimental difficulty of keeping E constant is great, unless a very special resistance or potential divide is employed. Consequently we have the following equations:

If $$E = n_2 \frac{dN}{dt}, \text{ and } \frac{dN}{dt} = k$$

being the secondary turns, then since the total change of flux is

$$\int_{t_1}^{t_2} \frac{dN}{dt} dt = k(t_2 - t_1),$$

and since E in volts $= \frac{kn_2}{10^8}$, the total change of flux is

$$N_2 - N_1 = k(t_2 - t_1) = \frac{E10^8}{n_2}(t_2 - t_1).$$

Hence by observing current and time a magnetisation curve can be deduced when the E.M.F. is kept constant in each case. For further details of the method which offers some advantages, the original paper must be consulted.

Alternating current methods are sometimes used depending on a formula such as

$$B = \frac{E}{N \, . \, n \, . \, A} \times \frac{1}{4k} \times 10^8,$$

where E is the R.M.S. value of the induced voltage in the secondary coil,

N the number of secondary turns,
n the cycles per second,
A the cross sectional area of the ring in cms.,
B the maximum flux density,
k the wave form factor, being 1·11 for a wave of sine shape.

The maximum value of the magnetising current may be obtained by multiplying the R.M.S. by the form factor for current, which has to be experimentally determined.

Messrs. Morris & Langford, in the paper above referred to, give the following comparison of the percentage accuracy of the different methods :

B =	1000.	2000.	3000.	4000.	5000.	6000.	7000.	8000.	9000.
Uniformly varying flux . . .	+3·2	+1·6	+0·7	0·0	−0·2	+0·7	+0·9	+0·2	+0·9
Step by step . .	..	−2·0	..	−5·7	..	−2·3	..	−0·7	..
Reversals . .	..	+6·2	..	+2·0	..	−0·5	..	+0·3	..

As regards accuracy, we have to consider the accuracy in **H** and accuracy in **B**. It may be mentioned that the chief sources of inaccuracy in testing by some otherwise accurate method are heating of specimen by magnetising coil causing annealing to occur and magnetic viscosity causing creeping.

As regards traction methods of testing, Dr. Taylor

Jones in testing the law of pull, viz. $\dfrac{\mathbf{B}^2}{8\pi}$ per unit area, found the values $\frac{1}{2}$ per cent above the theoretical for values up to 19,000 C.G.S. of the induction, and 1 per cent low for higher inductions. This was for polished surfaces pulled apart (*Phil. Mag.* vol. xxxvi. p. 354, 1895), the planes of the surfaces having been optically tested.

Sometimes a bar and yoke method of testing is used, the bar lying on the yoke as in Fig. 131.

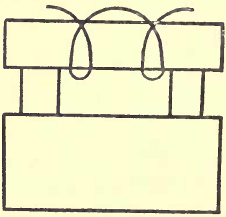


Fig. 131.—Bar and yoke.

In such a case several methods are open to us. We may either magnetise the specimen to a certain amount and then reverse the current, or magnetise it, then break the current circuit, then lift the yoke and note the ballistic throw, and, again, note the residual throw on replacing the bar. The sum of the three throws in the latter case will give the magnetic induction in the bar, the yoke residual being negligible. When such methods are compared, we obtain something like the following:

Specimen tested of Mild Steel

Reversal method .	1	2	11·5	13·25	19·5	22·25	25	26
Residual break .	8·5	9·5	10·5	14·5	16·5	21·5	26·5	27

The deflections are those of a fluxmeter. At the low inductions there is a considerable discrepancy between

the two methods, but as the induction approaches higher values the agreement is closer. This specimen was of mild steel; with soft wrought-iron the agreement is much closer between the two methods.

The fact seems to be that the coercion force in steel prevents the magnetic inductions, or change over, rising to an equal and opposite negative value. This is also noticeable in using the Friction Permeameter with a hard cast-iron specimen. Sometimes on reversing current after measuring the friction pull, the friction pull was reduced to zero. This was always at low inductions. See also *Magnetic Induction in Iron and Other Metals*, p. 65.

For general ballistic work the double yoke method of Ewing seems incomparably superior to all other methods when the material is in the form of rods, since in this case all error due to joints is eliminated.

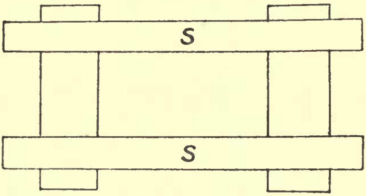


FIG. 132.—Double bar and yoke.

In this case the joints in soft materials of high permeability would cause an error, but this is eliminated by altering the length of the bars to half their original length. Hence we have the following equations:

$$H = H' - \frac{\epsilon}{L} \text{ in first test.}$$

$$H = H'' - \frac{2\epsilon}{L} \text{ in second test.}$$

$$\therefore \quad H'' - H' = \frac{\epsilon}{L},$$

where ϵ is the error due to the joints; consequently, by taking a set of reading of **B**, and apparent **H** for the two cases, and subtracting them horizontally, the true **B**—**H** curve is found. Care has to be taken to completely demagnetise the bar by beginning another set of experiments, otherwise conflicting results may be obtained.

Hysteresis Testing.—This may be considered from two points of view. If a hysteresis loop is available from ballistic or other tests, then a measurement of the area gives the value of the integral

$$\frac{1}{4\pi}\int \mathrm{H}d\mathrm{B}$$

or the loss per cycle in ergs per cubic centimetre.

More directly it may be tested by a hysteresis tester as devised by Ewing.

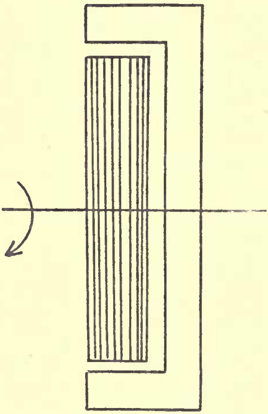


FIG. 133.—Ewing Hysteresis Tester.

A bundle of stampings, made thin to eliminate eddy currents, is rotated by hand between the poles of a C-shaped permanent magnet.

The permanent magnet being arranged so that it can deflect, the torque can be measured.

In this case, if n is the revolutions per minute, T the torque, then

$$2\pi n\mathrm{T} = \frac{n}{4\pi}\int \mathrm{H}d\mathrm{B}.$$

But T is $= k\mathrm{M}g \sin\theta.$

If the angle is small where k is a constant,

$$\sin \theta = \theta.$$

Hence $$\theta \propto L,$$

where L is the loss per cycle. It appears that if the speed is not too high the torque due to fanning is negligible.

In reality, however, the deflection varies, as pointed out by Ewing, as

$$kL + \text{constant},$$

the constant arising from the fact that in a sample without hysteresis the alteration of flux through the magnet would set up a torque. For absolute purposes the instrument requires calibration.

In such a method of testing the field strength or magnetic induction through specimen is always constant, owing to the relatively large air gap. It, however, does not permit of testing with a varying induction above that limit.

For general testing, the method adopted by the Bureau of Standards at Washington and described by L. W. Wild (*Jour. I.E.E.* vol. xlvi. p. 217) may be used. The stampings of transformer iron are built up into a square (see Fig. 134) with interleaved corners. There are four magnetising bobbins.

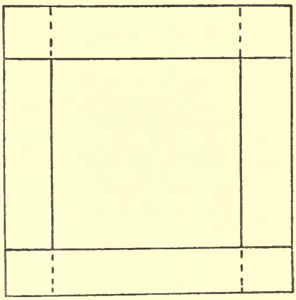


FIG. 134.—Method of assembling stampings.

To minimise instrumental errors, the voltage current

and CR drop were made practically the same (see vol. xlvi. p. 220).

The diagram shows the connections:

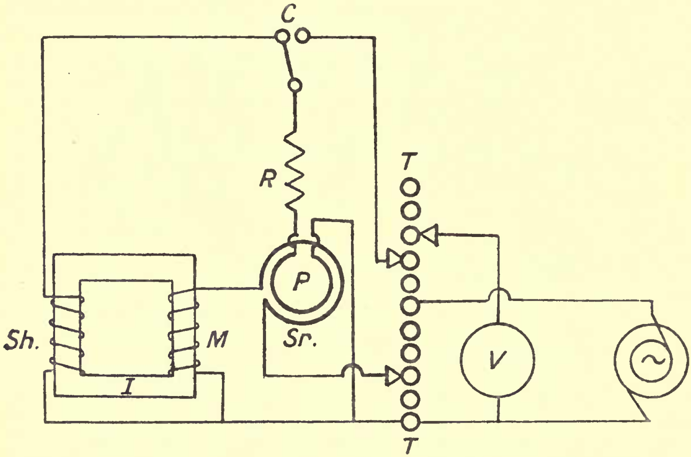


FIG. 135.

I, Iron under test.
M, Magnetising coil.
P, Pressure coil of wattmeter.
Sr, Series coil of wattmeter.
R, Non-inductive resistance, variable to alter range of wattmeter.
T, Eleven terminals of an auto-transformer, with ten equal windings.
V, Electrostatic voltmeter.
C, Two-way switch.
S*h*, Shunt winding on iron.
∾ Alternator.

There are two objections to such a method of testing, viz. that the difference between the longest and shortest flux path is considerable, and secondly, at the corners, the cross section being the diagonal of a square, is $\sqrt{2}$ times the normal. Sometimes pieces are interposed rounding the corners.

Regarding the form factor in such tests, an Original Communication, by L. W. Wild (*Jour. I.E.E.* vol. xliv. p. 222), may be consulted. By means of three sets of

windings, and transformers to alter ratios, hundreds of wave forms may be produced.

Also in an Original Communication to the same body (vol. xliii. p. 553) Albert Campbell gives a method of obtaining the total loss at standard frequency and wave form. This is referred to in the chapter on Wave Form (*vide* vol. ii.).

For small thicknesses of sheets, the eddy loss is proportional to the square of the thickness, but for thicker sheets it has been shown by J. J. Thomson to vary, as

$$\frac{\sinh 2ma - \sin 2ma}{\cosh 2ma + \cos 2ma}$$

where $2a$ is the thickness, and

$$m = 2\pi\sqrt{\mu n / \rho}.$$

μ is supposed the "constant" permeability, n the frequency, and ρ the specific resistance.

In making up rings of stampings to test for hysteresis or magnetic induction, it must not be forgotten that

$$H = \frac{4\pi n C}{10 l}.$$

Here $$l = \pi D,$$

$$\therefore \quad H \propto \frac{1}{D},$$

where D is the diameter of the ring.

The ratio actually measured is the mean of $(B)_{max}^{1 \cdot 6}$, whereas it should be proportional to the

$$(\text{mean } B_{max})^{1 \cdot 6}.$$

By having an H—B curve the mean $B^{1\cdot6}_{max}$ over the cross section can be found, and the ratio of $\frac{(\text{mean } B^{1\cdot6}_{max})}{B^{1\cdot6}_{max}}$ (the denominator corresponding to the observed loss) gives the correcting factor (see paper by Campbell referred to above).

Stamping out rings increases the hysteresis loss, owing to the hardening of the edge. In cases of strips 3 cm. wide, this increases the loss by 12 per cent over that of a strip infinitely broad. This assumes that the effect of stamping hardens the strips an equal distance inwards in each case.

L. W. Wild also states that "it is generally agreed that if accurate results are to be obtained the strip must be annealed after being cut. Strip 10 inches by 1 inch wide are recommended in this method.

Dr. Kapp in an Original Communication to the Institution of Electrical Engineers (*Jour. I.E.E.* vol. xxxix. p. 227) gives the following method for plotting hysteresis loops for iron.

Let Z be the flux in megalines produced by a continuous current of I_0 amperes through n turns of a winding having e volts applied to it, then

$$e = RI_0,$$

where R is the resistance of the coil.

Let e now be reversed, then I will change from $-I$, its initial value through zero, to $+I$, its final value.

At any instance the current must satisfy the equation

$$e = \frac{n}{100} \frac{dZ}{dt} + RI.$$

By observing t and I a time current curve may be plotted, and from this curve and the known values of e and n a hysteresis loop giving z as a function of I may be drawn.

This method is applicable to the iron of transformers or choking coils, provided the time taken by the current to rise to a maximum is not too short.

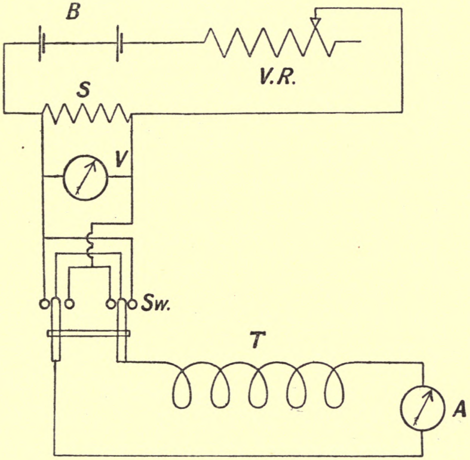


FIG. 136.—Kapp's method of plotting hysteresis loop.

For a 10-kilowatt transformer it is about 4 seconds, $6\frac{1}{2}$ with a 20-kilowatt, and 16 seconds with an 80-kilowatt transformer. The speed of the needle is about proportional to the $\frac{2}{3}$ power of the output.

The arrangement is as shown in Fig. 136.

T is the coil to be tested, A an ammeter with central zero, V the voltmeter reading across the shunt S, S*w* is the change over switch, V.R. a variable resistance, and B a battery.

To make the test, adjust V.R. till I_0 is read on the

ammeter A. Then change over switch and read current, and at the same time note the time by means of a stop-watch. The time current curve would be truly logarithmic provided there was no hysteresis loss, but hysteresis causes it to have the form shown:

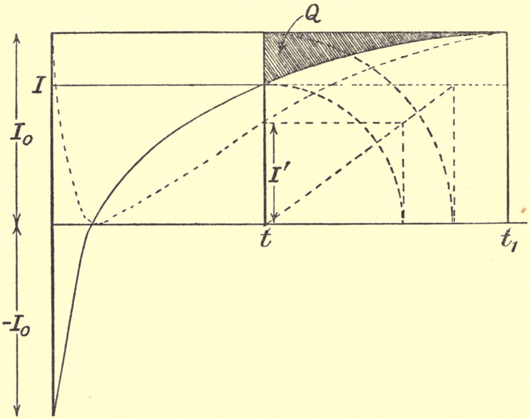


FIG. 137.—Kapp's method.

We might write

$$\frac{100}{n}(e - \mathrm{RI}) = \frac{d\mathrm{Z}}{dt},$$

or
$$\frac{100\mathrm{R}}{n}(\mathrm{I}_0 - \mathrm{I})dt = d\mathrm{Z},$$

since $\mathrm{I}_0 - \mathrm{I}$ is the distance between point on curve and the asymptote I_0, so that

$$\int(\mathrm{I}_0 - \mathrm{I})dt$$

is the area marked shaded on the figure integrating between limits $-\mathrm{Z}$ and $+\mathrm{Z}$.

$$2Z_0 = \frac{100R}{n} Q_0 \quad . \quad . \quad . \quad (1)$$

if by Q_0 we denote the whole area between the curve and its asymptote.

Integrating for any other flux Z we have

$$Z_0 + Z = \frac{100R}{n}(Q_0 - Q) \quad . \quad . \quad (2)$$

combining (1) and (2) we have

$$Z = \frac{100R}{n}\left(\frac{Q_0}{2} - Q\right) \quad . \quad . \quad (3)$$

or

$$Z = \frac{100e}{nI_0}\left(\frac{Q_0}{2} - Q\right) \quad . \quad . \quad (4)$$

Hence, if we fix on a value of I, find by means of a planimeter the corresponding area Q, and from (4) the value of the induction.

The values of I and Z may then be plotted as a hysteresis loop, and we see the hysteresis loss per cycle is

$$E = \frac{n}{100} \times (\text{area of loop}).$$

When joints are present the instantaneous current is less than I. It may be found by the graphical construction shown.

The hysteresis loss for a cycle is now

$$E = 2e\int_0^{t_1}(I - I')dt \text{ watt seconds},$$

or

$$E = 2eQh.$$

Very similar methods of testing had previously been pointed out by Messrs. Morris & Lister in their paper (*Jour. I.E.E.* vol. xxxvii. p. 283). It is as follows:

In the primary of a transformer

$$E_1 i_1 = e_1 i_1 + i^2 r \text{ watts during any period,}$$

$$\therefore \quad \int E_1 i_1 dt = \int e_1 i_1 dt + \int i^2 r dt = \text{magnetic energy} + \text{ohmic losses.}$$

If e_1 is kept constant $= ke_2$, where k is ratio of transformation in the transformer, magnetic energy is

$$Ke_2 \int i_1^2 dt.$$

Then plot i_1 against time for rising and falling currents. See p. 28a of the paper referred to.

The great advantage of such methods is the simplicity, merely reading an ammeter and noting the time in order to obtain the $I - t$ curve.

By superposing direct current magnetisation on an alternating flux, M. Rosenbaum, B.Sc. (*Jour. I.E.E.* vol. xlviii. p. 535, "Hysteresis Loss on Iron taken through Unsymmetrical Cycle of Constant Amplitude"), found that the hysteresis loss is increased in such cases, and refers to the action of a pulsating flux in inductor alternators and static balancers.

Calorimetric methods of testing iron have not been greatly in vogue. They do not appear to admit of such accuracy as wattmeter, or the B—H loop method. Nevertheless, they may be improved upon later. The first evaluation of $\frac{1}{4\pi}\int H dB$ was by means of the calorimetric method by Professor Ewing and Miss Klaasen, when the heating of two small transformers was compared.

Standard Fields

For calibrating a ballistic galvanometer or fluxmeter, a standard solenoid is often used. If a search coil consists of n turns of wire enclosing an area A, then if the strength of field is H lines per square centimetre, H An is the number of lines cut on switching off current, or 2HAn on reversing.

When a search coil is wound on a bobbin and placed inside a standard solenoid at its centre, allowance must be made for the different areas of the various layers of wire,

$$nA = n_1 A_1 + n_2 A_2 +, \text{ etc.},$$

where n_1 is the number of turns on layer of area, A_1, n_2, n_3, A_2, A_3 the turns and areas of the other layers. The same thing applies if a coil of n turns is wound over a primary.

On reversing, as is usually done in calibrating, we see that the lines per unit angles or divisions is given by

$$2\text{H}an = k\theta$$

or

$$k = \frac{2\text{H}an}{\theta}$$

for a given instrument.

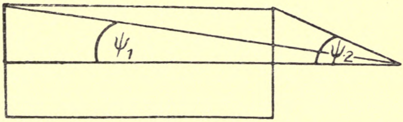


Fig. 138.—Field along axis of solenoid.

The magnetic force H at any point on the axis of a solenoid is given by

$$\text{H} = 4\pi n\text{C}(\cos\psi_1 - \cos\psi_2),$$

that is, $\text{H} = 4\pi n\text{C}$ at its centre.

Expressions for the magnetic force at points distant y from the axis will be found in Gray's *Absolute Measurements on Electricity and Magnetism*, p. 253).

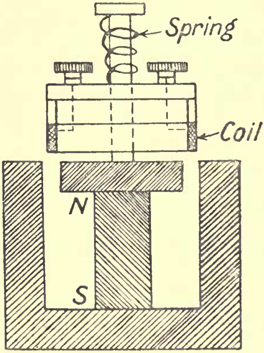


FIG. 139.—Standard Magnet.

Standard Magnet.—These are sometimes used for calibration purposes and consist of a permanent magnet aged sufficiently so that the field is invariable. A coil is arranged to cut a narrow gap so that nN gives a throw on the ballistic galvanometer.

Flux Measurement.—Frequently it is necessary to investigate the distribution of magnetic flux under poles of dynamos or motors, and for this purpose the following methods are used :

1. Ballistic galvanometer or fluxmeter and search coil.
2. The bismuth spiral.
3. The Morphy and Oschwald Fluxmeter.

The ballistic galvanometer and also the fluxmeter are sufficiently well understood, and have already been dealt with in chapter on Damping.

The bismuth spiral is merely a strip of bismuth which possesses the extraordinary property of altering its resistance when subjected to magnetic flux. If the magnetising coil getting hot radiates heat on to the bismuth, the results are very discordant, as not only does the resistance change on account of temperature coefficient, but effect of magnetic field is altered. See *Handbook for the Electrical Laboratory*, by J. A.

Fleming, M.A., D.Sc., F.R.S., vol. ii. p. 406, fig. 2. Otherwise, if inserted in various field strengths, it has corresponding resistances. A calibration curve is necessary when using it, and this is by no means a straight line. It is also insensitive at low readings.

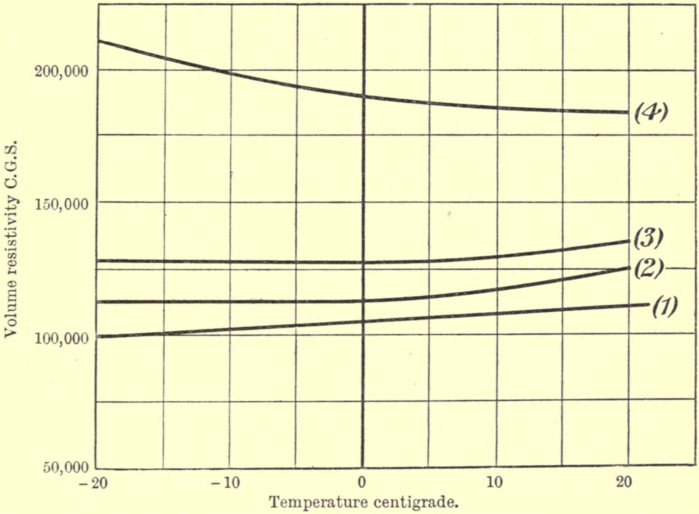


FIG. 140.—Volume resistivity of bismuth.

(1) Out of field. (2) In field of 2450 C.G.S. (3) 5500 C.G.S. (4) 14,200 C.G.S.

THE MORPHY AND OSCHWALD FLUXMETER

The Morphy and Oschwald fluxmeter (*vide Electrician*, Jan. 19, 1912) is interesting as a fluxmeter for exploring fields of force in narrow gaps, such as that around the field magnets of dynamos or motors.

The essential principle involved is that the magnetic flux is measured by noting the deflection of a very narrow

coil traversed by a current which is kept constant while the coil is immersed, so to speak, in the magnetic field.

The instrument will be understood from Figs. 141, 142.

The instrument consists essentially of a slender shaft (0·032 in. brass), with needle points at each end, carrying a coil of 20 turns of No. 47 S.W.G. copper wound longitudinally between two notches. The shaft is flattened

Fig. 141.—Movement (not to scale).

H, Hair spring. M, Mirror. L, Ligaments.

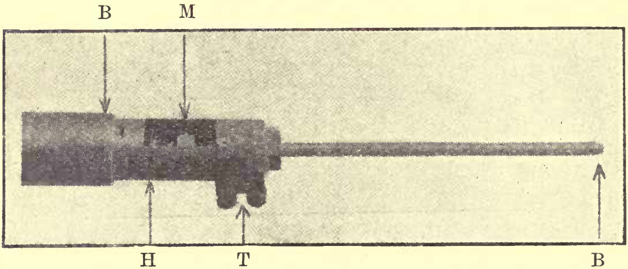


Fig. 142.—Movement in case.

T, Terminals. H, Hair spring.
B, B, Bearings. M, Mirror.

by filing before being wound, so that flat bases are formed for the coil to lie upon. The ends of this coil are connected through light silver ligaments (L, Fig. 141) to terminals (T, Fig. 142). The shaft also carries a fine hair spring (H) and a square brass block, on the 4 sides of which small mirrors (M) are mounted. The whole movement is mounted between jewelled bearings (B, Fig. 142) inside brass tubing, $2\frac{3}{4}$ in. of which have a

diameter of $\frac{1}{8}$ in., and the remainder (which surrounds the mirrors, hair spring and ligaments) a diameter of $\frac{5}{8}$ in.

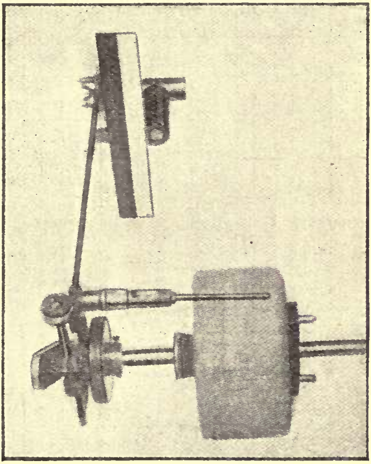


FIG. 143.—Apparatus and armature.

Windows are cut in the larger part of the brass tube so that light may fall from a small Osram lamp on to the mirrors, after reflection from one of which the beam may be caught upon a scale (Fig. 143).

The large brass tube is terminated by a socket which makes a good sliding fit with a projecting cylinder on the arm (Fig. 143). This arm is constructed so that an extension piece, which carries the lamp and scale, may be securely clamped either at right angles to, or in continuation of, the line of the arm. The former of these two positions is shown in Fig. 143, and they are intended to allow the beam of light to be cast on to whichever of the mirrors is most convenient for any particular position of the movement. It also admits of the avoidance of any obstacle which might interfere with the path of the beam. The distances from the lamp and scale to the mirrors were the same in each case, and approximately $7\frac{1}{2}$ in.

The position of the arm and movement relatively to the armature is read on a circular scale and adjusted by

means of the worm and wheel, which can be seen in Fig. 143.

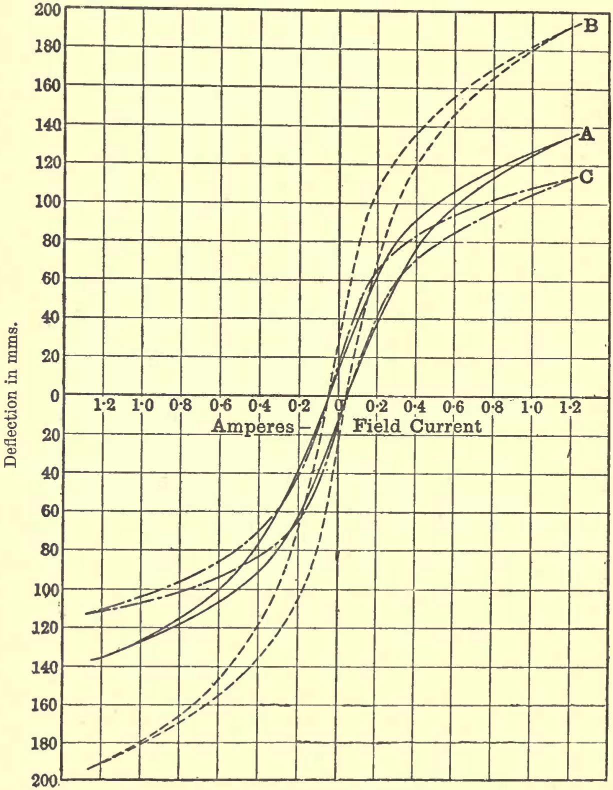


FIG. 144.—Hysteresis loops.

Coil current	0·027 amp.
Armature current	Zero.
Smooth core armature	Full line curve A.
Slotted armature (movement over tooth) . .	Dotted curve B.
Slotted armature (movement over slot) . .	Chain dotted curve C.

The armature of the experimental machine, on which

the instrument is mounted, can be turned through 180 deg. by hand, but is not intended to run. During operation it is clamped in a definite position, which can be noted relatively to a second circular scale, fixed to the frame on the side remote from the movement.

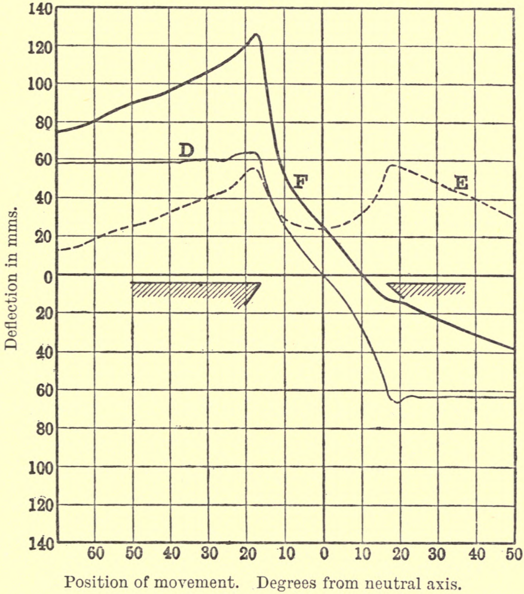


FIG. 145.—Flux distribution.

Smooth core armature. Commutation axis coinciding with neutral axis. Current through movement 0·027 amp.

Curve D.—Field, 0·3 amp. ; Armature, 0 amp.
Curve E.—Field, 0·0 amp. ; Armature, 15 amp.
Curve F.—Field, 0·3 amp. ; Armature, 15 amp.

When a current is passed through the small coil, which is primarily set so as to lie in a place normal to the armature surface, it tends to turn into a plane normal

to the flux in which it is situated. This motion is opposed by the hair spring, and the coil, of course, takes up a position in which the opposing moments are equal.

Operation.—In practice a current of the order of 0·03 ampere is passed, first in one direction and then in the reverse, through the coil, and the deflection of the reflected beam of light on the scale noted in each case. The sum of these two deflections, in units of length, may, for most purposes, be assumed to be proportional to (1) that component of the flux which is normal to the armature; (2) to the current passing through the movement. If great accuracy be required a correction curve may be calculated from the formulae given under the heading of "Theory," where it is shown that the maximum error introduced by the first-mentioned assumption will not exceed 2 per cent with a deflection of 120 mm.

By means of this apparatus, not only can hysteresis loops be readily obtained from the machine, but the effects of teeth in modifying the flux distribution can be studied.

The method possesses the advantage that by altering the current strength through the moving coil, portions of a flux distribution may be studied with great minuteness and accuracy.

The theory is simple and is as follows:

Theory

Symbols.—

θ_1, θ_2 Angular displacement of coil from zero position on either side of zero.

d_1, d_2 Corresponding linear displacement of spot of light along scale (in mms.).

α Angle between direction of flux and initial plane of coil (normal to armature).

B Induction in lines per square cm.
C Current through movement coil in amperes.
l Length of movement coil in cms.
b Breadth of movement coil in cms.
a Area of movement coil in sq. cms.
n No. of turns in movement coil.
L Perpendicular distance from mirror to scale (mms.).

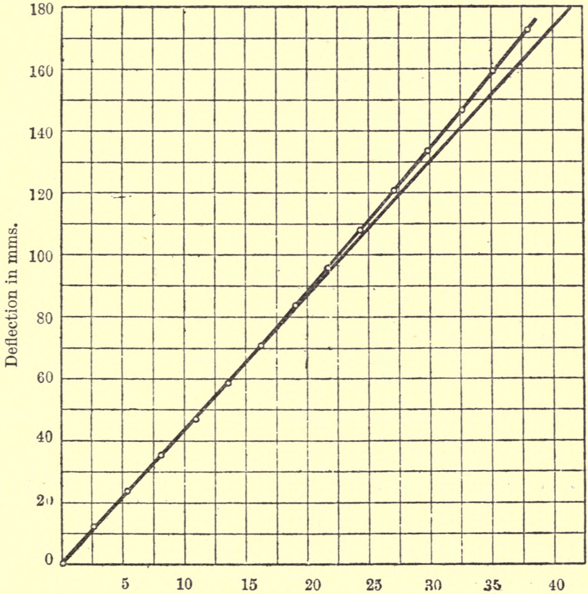


FIG. 146.—Calibration of movement.

When coil is deflected through an angle θ,

$$\text{Deflecting torque} = 2n \cdot \frac{CB}{10} \cdot l \cdot \frac{b}{2} \cdot \cos(\theta \pm a)$$

$$= \frac{na}{10} \cdot CB \cdot \cos(\theta \pm a).$$

Restoring torque $= k_1\theta$ dyne cms. where k_1 is the constant of the spring.

$$\frac{na}{10} \cdot \text{CB} \cdot \cos(\theta \pm a) = k_1\theta,$$

or
$$\text{CB} \cdot \cos(\theta \pm a) = k\theta$$

where
$$k = k_1\frac{10}{na} \quad . \quad . \quad . \quad (1)$$

Equation (1) is the true law for the instrument and relative values of B cos a may be determined by taking readings of d_1 and d_2 upon the scale and converting them into corresponding degrees of deflection from the equations

$$d_1 = \text{L} \cdot \tan 2\theta_1$$
and
$$d_2 = \text{L} \cdot \tan 2\theta_2 \quad . \quad . \quad . \quad (2)$$

Rewriting equation (1),

$$\text{CB} \cos(\theta_1 - a) = k\theta_1$$
$$\text{CB} \cos(\theta_2 + a) = k\theta_2$$

$$\text{CB} \cdot 2 \cdot \cos\left(\frac{\theta_1 + \theta_2}{2}\right) \cdot \cos\left\{a - \left(\frac{\theta_1 - \theta_2}{2}\right)\right\} = k(\theta_1 + \theta_2) \quad (3)$$

also
$$d_1 + d_2 = \text{L} \tan 2\theta_1 + \text{L} \tan 2\theta_2$$

combining these two.

$$\text{CB} \cdot \cos\left(a - \frac{(\theta_1 - \theta_2)}{2}\right) = \frac{k(\theta_1 + \theta_2)}{2\text{L}(\tan 2\theta_1 + \tan 2\theta_2) \cos \frac{(\theta_1 + \theta_2)}{2}} \times (d_1 + d_2).$$

The term

$$\frac{k \cdot (\theta_1 + \theta_2)}{2\text{L}\,(\tan 2\theta_1 + \tan 2\theta_2) \cdot \cos\frac{(\theta_1 + \theta_2)}{2}}$$

is constant to within 2 per cent when θ does not exceed 9 deg., which is the limit for ordinary work upon the instrument, hence

$$\mathrm{CB} \,.\, \cos\left(\alpha - \frac{\theta_1 - \theta_2}{2}\right) = k_2(d_1 + d_2) \qquad . \quad (4)$$

where $$k_2 = \frac{k(\theta_1 + \theta_2)}{2\mathrm{L}\,(\tan 2\theta_1 + \tan 2\theta_2) \cos \frac{\theta_1 + \theta_2}{2}}.$$

Also it can be shown that

$$\tan \alpha = \frac{\theta_1 - \theta_2}{2\theta_1\theta_2} \text{ approximately } . \qquad . \quad (5)$$

θ_1, θ_2 being expressed in radians,

or $$\frac{\theta_1 - \theta_2}{2} = \theta_1\theta_2 \tan \alpha.$$

The usual limit of θ_1, θ_2 was about 9 deg. and except in the interpolar space α did not exceed 30 degrees.

Hence the value of $\frac{\theta_1 - \theta_2}{2}$ for these conditions is given by

$$\frac{\theta_1 - \theta_2}{2} \text{ (radians) is} < \left(\frac{9\pi}{180}\right)^2 \tan 30 \text{ deg.}$$

and $$\frac{\theta_1 - \theta_2}{2} \text{ degrees is} < \frac{81\pi}{180} \frac{1}{\sqrt{3}},$$

i.e. $< 0{\cdot}8$ of a degree.

Equation (4) may be written

$$\mathrm{CB} \,.\, \left(\cos \alpha \,.\, \cos \frac{\theta_1 - \theta_2}{2} + \sin \alpha \,.\, \sin \frac{\theta_1 - \theta_2}{2}\right) = k_2(d_1 + d_2)$$

minimum value of $\cos \frac{\theta_1 - \theta_2}{2} = 0{\cdot}9999,$

maximum value of $\sin\frac{\theta_1-\theta_2}{2}\sin a = 0{\cdot}014 \times \frac{1}{2} = 0{\cdot}007$,

hence if we write $\cos a \,.\, \cos\frac{\theta_1-\theta_2}{2} = \cos a$,

and neglect the term $\sin a \,.\, \sin\frac{\theta_1-\theta_2}{2}$ the error is less than 1 per cent.

In the interpolar space a is generally greater than 30 deg., but at the same time the values of θ_1 and θ_2 decrease as a increases, so that $\sin a \,.\, \sin\frac{\theta_1-\theta_2}{2}$ is always negligibly small.

Thus we can always write

$$\cos a \text{ for } \cos\left(a - \frac{\theta_1-\theta_2}{2}\right),$$

and (4) becomes $\quad \mathrm{CB} \,.\, \cos a = k_2(d_1+d_2) \quad . \quad . \quad (6)$

Calibration of Movement.—Relation between current through movement and the deflection produced, the flux being constant.

This test was carried out with the maximum value of a that occurs under the poleface, and yet the curve obtained is seen by Fig. 146 to be a straight line for values of d_1+d_2 up to 80 mms., confirming the relation

$$\mathrm{CB} \,.\, \cos a = k(d_1+d_2) \quad . \quad . \quad (7)$$

The value of B corresponding to a given value of C and (d_1+d_2) was approximately obtained by placing the coil of a Grassot fluxmeter in a part of the air-gap when the flux was uniform and with its direction normal to the surface of the armature, and noting the reading on making or breaking a certain field of current. This reading was 97 scale divisions, each division correspond-

ing to 17·45 lines per square centimetre. Hence the change in induction (maximum less remanent) is $17{\cdot}45 \times 97$ lines per square centimetre.

The change in the value of $(d_1 + d_2)$ for the same change of flux was 119 mm., the current through the movement being 0·027 ampere.

Substituting in (6)

$$k = \frac{\mathrm{CB} \cos a}{d_1 + d_2} = \frac{0{\cdot}027 \times 17 \,.\, 45 \times 97}{119} = 0{\cdot}384.$$

Hence CB . cos $a = 0{\cdot}384(d_1 + d_2)$ for the particular instrument under test, where :

C = Current in amperes through movement.
B = Lines per square centimetre.
d_1, d_2 = mm. deflection with reversal of C.
a = Angle between the flux and the normal to the armature.

Another somewhat interesting but rather more laborious method of using the apparatus is to start with the coil initially deflected from its zero position, and then adjust the coil current till the coil is brought to zero again.

In this case we see that

$$\mathrm{CB} \cos a = k\theta,$$

so that

$$\mathrm{B} \cos a = \frac{k\theta}{\mathrm{C}},$$

where θ is the initial deflection and CB and k have the same values as previously. We see from above that if θ is constant

$$\mathrm{B} \cos a \propto \frac{1}{\mathrm{C}}.$$

As a further alternative method C might be kept constant and the coil turned through an angle θ to bring the spot to zero. In this case

$$B \cos a \propto \theta.$$

This latter method is, however, troublesome in practice without some fine adjustment to bring the spot to zero. The angle could be easily measured by deflection of the spot of light, but since the movement is fixed friction tight to the arm, it moved in jerks, and it was difficult to set the spot exactly to zero.

CHAPTER X

THE POST OFFICE BOX

Of all testing instruments the one most frequently used for comparing resistance is the Post Office Box. The connections of the instrument are shown in Fig. 147. As a rule, little attention is paid nowadays to the rules for the best manner in which to connect battery and galvanometer, as given in Clerk Maxwell's *Electricity and Magnetism*, and physical text-books.

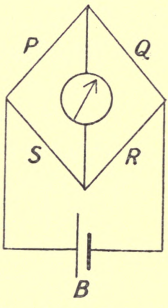


Fig. 147.—Diagram of Post Office Box.

Generally speaking, if the galvanometer has a greater resistance than the battery, it should be connected between the junction of the two *least* resistances and junction of the two greatest. When the battery has the greater resistance, the *battery* is inserted in this position.

This rule was probably of more importance in the days when galvanometers were not very sensitive, but with the D'Arsonval types of instruments now invariably used, they are of comparatively little importance.

The best resistance of the galvanometer is of more

importance, and if we denote the ratio arms by P, Q, R, S, and let

$$P + Q = A,$$
$$R + S = B,$$

then the best resistance of the galvanometer is that which makes it

$$= \frac{1}{\frac{1}{A} + \frac{1}{B}}.$$

This latter rule enables one to say whether it is better to use a high or low resistance galvanometer when a given resistance is being measured (see Schwendler, "Galvanometer Resistance to be employed in Testing with Wheatstone's Diagram," *Phil. Mag.*, May 1866).

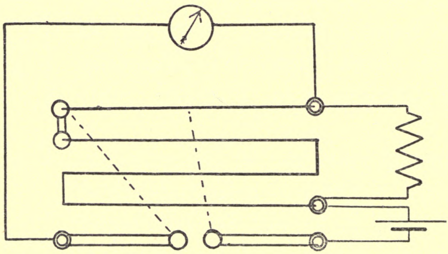


FIG. 148.—Connections of Post Office Box.

It is easily shown (*vide* Stewart and Gee, *Elementary Practical Physics*, vol. ii., or Fleming's *Laboratory Handbook*) that the current through G due to any battery of E.M.F. = E for any given disposition of P, Q, R, S, is

$$i = \frac{E(QS - PR)}{D}.$$

D is a determinant which, besides involving these resistances, involves also the resistance of the battery and galvanometer. It is apparent, therefore, that if

$$QS = PR,$$

the current through the galvanometer is zero, so that we have

$$\frac{P}{Q} = \frac{R}{S},$$

or the " law " of the Wheatstone Bridge.

In using the Post Office Box " balance " is said to be obtained when after closing the battery key and then sharply tapping the galvanometer key no deflection is obtained.

For the purpose of bridge testing, we ought to use a galvanometer which is most sensitive about the zero position. That is to say, if it is a D'Arsonval the coil should, near the zero position, be moving in the strongest portion of the field due to the permanent magnet, hence chamfered poles.

Sometimes in testing, it is impossible to obtain a zero reading, since an ohm more or an ohm less will cause the galvanometer needle to deflect, say, from the right to the left. In such a case it is usually sufficient to interpolate by proportion. Suppose the resistance being measured was ca. 1·00 ohm, but the galvanometer spot of light was 10 to the left of zero. On taking out 1 ohm it goes 30 to the right. Hence 1 ohm gives 40 divisions, therefore 10 divisions = ·25 ohm, and the result is 1·25 ohms, provided the ratio arms are equal.

When starting to test a resistance, it is preferable to keep the ratio arms of the bridge equal and balance

roughly, using a not too sensitive galvanometer. One can then see at once between what limits the resistance under test lies. Then the ratio arms may be altered and a more sensitive galvanometer used and further decimal places obtained.

The Post Office Box was formerly used to test insulation, but in engineering practice, insulation should be tested at the working voltage at least, and it is preferable to use a "Megger."

The ratio arms being 10, 100, 1000, then with 1 ohm out in the resistance box and 10 in other ratio arm, the lowest resistance capable of measurement is

$$\frac{10}{1110} \text{ or } \frac{1}{111} \text{ ohm.}$$

In the bridge there is an "infinity" plug. If the resistance being tested is very high, it was customary to pull out the infinity plug, and then on tapping down the galvanometer key, and obtaining no deflection, the resistance tested was called "infinite," the meaning, of course, being that the resistance was much higher than anything the bridge could tackle.

If the battery and galvanometer are joined up as in P, Q, R, S above, it is quite clear that if both R and S are infinite no current can pass through the galvanometer, and the method of test with infinity plug simply breaks circuit, and is quite meaningless.

Kelvin Double Bridge

For comparing extremely low resistances this method may be used, and by means of it the resistance of joints

in a large cable may be compared with a piece of the cable itself, or a piece of copper rod compared with a standard piece.

The following proof is given in Gray's *Absolute Measurements on Electricity and Magnetism*, the small

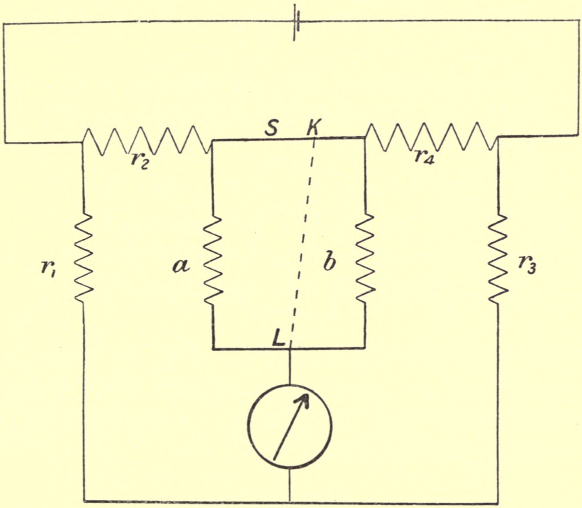


FIG. 149.—Kelvin Double Bridge.

book, p. 225. The student will find other proofs in Fleming's *Laboratory Handbook*.

a, b, r_1 and r_3 are all small resistances of about an ohm or so.

Suppose the point L connected with the same point K of s which is at the same potential as L. If balance is obtained by moving M till no current flows through the galvanometer, then the resistance of the

portion BC to the left of K is $\frac{as}{a+b}$, and to the right $\frac{bs}{a+b}$, the resistance now between B and KL is

$$\frac{a^2s}{a+b}\Big/ a+\frac{as}{a+b}, \text{ or } \frac{as}{a+b+s},$$

and similarly between C and KL,

$$\frac{bs}{a+b+s}.$$

Hence we have

$$r_3\left\{r_2+\frac{as}{a+b+s}\right\}=r_1\left\{r_4+\frac{bs}{a+b+s}\right\},$$

or $$r_1r_4-r_3r_2=\frac{s}{a+b+s}(ar_3-br_1).$$

If S is the portion connecting r_1 and r_4, the resistances being compared are small compared with a and b, and if ar_3-br_1 is small, then the quantity on the right will be approximately zero when

$$\frac{r_1}{r_2}=\frac{r_3}{r_4}.$$

It will be noticed if ar_3-br_1 is so adjusted as to be exactly zero, then the resistance of the connecting portion hardly matters, and may be as much as an ohm without affecting the sensitiveness.

The battery used should preferably be a storage cell with a resistance of one ohm or so in series with it to prevent the cell being short circuited. Use a resistance of such magnitude as not to run down the cell and to limit current to such a value as will not cause heating.

It can be shown (1) that the current through the galvanometer is independent of the resistance of the leads joining the resistances being compared for a small percentage want of balance.

Its value is

$$Z = \frac{C\{S(r_3 a - r_1 b) + (a + b + S)(r_2 r_3 - r_4 r_1)\}}{\Delta}$$

where C is a constant and Δ a determinant.

(2) The most suitable value for g is

$$g = \frac{2r_1 r_3}{r_1 + r_3}.$$

(3) The smaller r_1 and r_3, the larger the deflection for a given small percentage want of balance in the resistance.

(4) r_1 and r_3 must not be too low or else contact resistance will introduce error.

Tests made on a 61/14 *Cable.*—Wires were carefully soldered at *a*, *b*, *c*, *d*, and the resistance between these

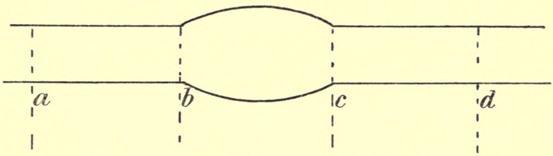


FIG. 150.—Test on jointed cable.

b, *c*, Joint.

points measured. Although the total resistance was only 20 microhms the resistances of the separate portions added together agreed to within 1 per cent.

Resistance of a Bar.—A cylindrical bar of brass about

36 cms. long, 0·25 cms. diameter was compared with a 0·01 ohm standard manganin strip resistance. The contact was measured by knife edges 30 cms. apart.

$$A = \cdot 01. \qquad B = \text{brass bar.}$$
$$P = R = 1000 \text{ ohms.}$$
$$Q = S \text{ between } 75\cdot 2 \text{ and } 75\cdot 3 \text{ for balance.}$$

$$B = \frac{75\cdot 2 \times \cdot 01}{1000} = 0\cdot 000752.$$

A change of 1 per cent in Q produced a readable change in the galvanometer deflection. The current was supplied by one secondary cell, E.M.F. 2 volts, and since it was passing through 1 ohm, the total current was about 2 amperes.

The Potentiometer

This is generally made in a variety of forms, viz. simple slide wire, direct reading, long range, thermo-electric and alternating current potentiometer.

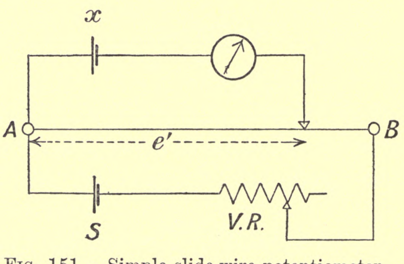


Fig. 151.—Simple slide wire potentiometer.

In the simple slide variety, as shown in Fig. 151, S is a battery of higher E.M.F. than that being tested, e is a standard cell in series with the galvanometer, and

V.R. a variable resistance. The method adopted is to pass current through the wire AB, and balance the E.M.F. of the standard cell against the volt drop on a portion of the wire. Suppose e is balanced by a length l. Substitute another cell x, whose E.M.F. is required in terms of that of the standard cell. Let it balance at l_1, then we have

$$\frac{x}{e}=\frac{l_1}{l}.$$

It follows that if we adjust the current from the cells by means of V.R. suitably, we can so arrange matters that the length l is proportional to e in scale divisions ; that is to say, if $e=1{\cdot}434$ volts, then we place the slider on 1434 scale divisions of the wire and adjust V.R. till balance is obtained. Consequently, if x balanced at 1070 divisions, its E.M.F. would be 1·07 volts.

This is the principle of all direct reading potentiometers.

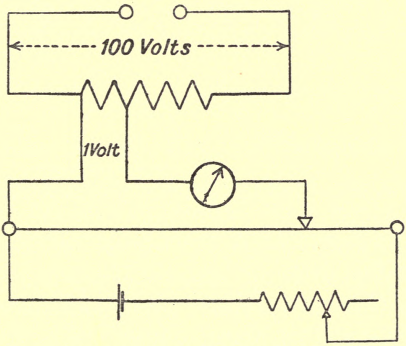


Fig. 152.—Measurement of high voltages by means of potentiometer.

High Voltages.—To measure high voltage continuous currents, these are passed through a multiple resistance,

and a fraction (known) of the total volt drop measured. The arrangement is as in Fig. 152.

In practice and for accurate work, coils carefully calibrated are used instead of a slide wire, and a series of terminals arranged to enable the various sources of

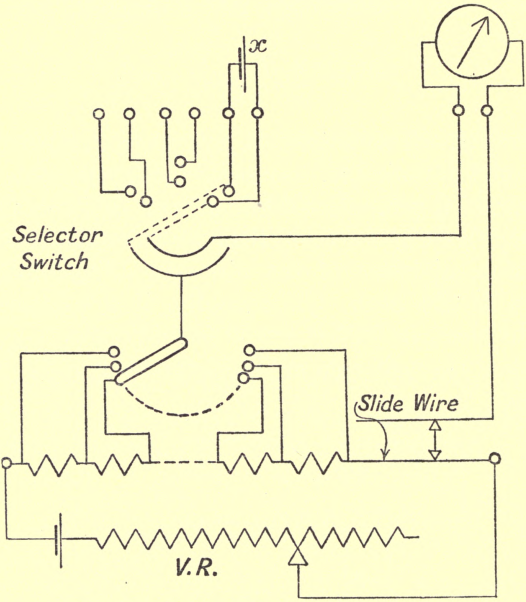


FIG. 153.—Commercial form of potentiometer.

E.M.F. to be tested to be rapidly connected up. This is shown in Fig. 153.

As a rule there is a slide wire for fine adjustment, seen on the right hand of above figure. This slide wire is sometimes a source of trouble, either by dirt collecting in it, or due to expansion owing to temperature changes. In the former case a little paraffin may be used to clean it.

The thermo-electric potentiometer of Carpenter-Stansfield is arranged so that an E.M.F. due to a thermo-junction may be balanced against that due to a standard cell. The current from the battery is passed through nine coils of 2·0 ohms and nine of 0·2 ohm and an E.M.F. of 0·2 micro-volt can be balanced, smaller differences being obtained from the galvanometer deflection (see

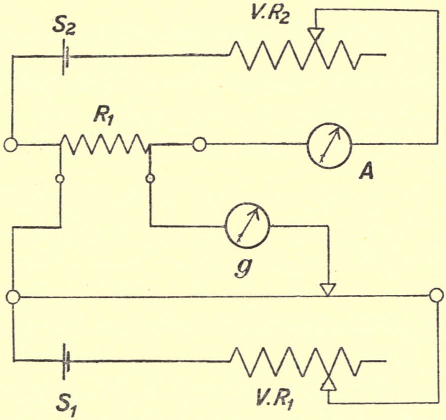


FIG. 154.—Calibration of an ammeter by means of a potentiometer.

also *Electrician*, Jan. 31, 1908, for description of Thermo-electric Potentiometer).

With an ordinary slide wire 1 or 2 metres long, it is quite possible to compare E.M.F.'s correctly to 1 per cent. For physical purposes, however, the large and carefully constructed potentiometers are capable of comparing E.M.F.'s to six significant figures.

It is quite clear that this forms by far the most accurate means of comparing E.M.F.'s or checking voltmeter and ammeter readings and resistances, since

standard cells are now obtainable with E.M.F.'s known to four or five significant figures.

It possesses also the advantage of being a zero method, all that requires to be done being alteration of the resistances till the spot of light is brought to zero. Consequently no great skill is required to obtain fair accuracy.

Ammeter Calibration.—Potentiometer made direct reading by means of standard cell in usual way.

Volt drop across R_1 measured $=l_1$, then $I=\frac{l_1}{R_1}$.

R_1 should be of such magnitude as to give about 1·5 volt drop with the maximum current employed, and of such gauge as not to heat up.

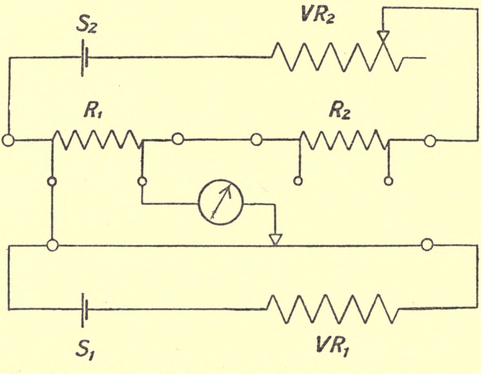


Fig. 155.—Comparison of resistances by means of a potentiometer.

Comparison of Resistances.—It follows that

$$\frac{e_1}{e_2}=\frac{l_1}{l_2}=\frac{IR_1}{IR_2}$$

$$=\frac{R_1}{R_2}.$$

High Voltages.—In the Hartmann and Braun Potentiometer high voltages are measured as follows :

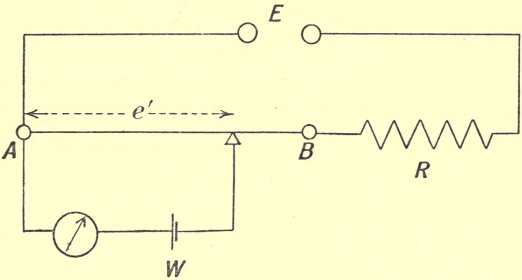


FIG. 156.—Measurement of high voltages by means of a potentiometer (Hartmann and Braun instrument).

The high voltage to be measured >10 volts <110 volts is connected as shown.

AB is the potentiometer proper. Total resistance, 10,000 ohms.

R is a plug box. Total resistance, 100,000 ohms—by steps of 1 ohm.

The standard cell W is placed across a suitable number of divisions, *e.g.* (1434 with Clark) $=1000\ e$, and balance is obtained by adjusting R. When balance is obtained the drop in 1000 *e* ohms is *e* volts, or current is ·001 ampere, if R be resistance in box.

Since resistance in potentiometer circuit 10,000 + R,

hence $$E=\frac{10{,}000+R}{1000}=10+\frac{R}{1000}.$$

THE ALTERNATE CURRENT POTENTIOMETER

This instrument, devised by Dr. C. V. Drysdale, resembles the continuous current potentiometer, the

principle of its action depending upon balancing any sinusoidal potential difference to be tested by a potential difference of equal magnitude and phase difference. By moving the potentiometer slides, the coils being of exceedingly small inductance and capacity, and turning the secondary of the phase shifting transformer, the phase of the potential difference is altered. Instead of the usual D'Arsonval reflecting galvanometer we must use either a vibration galvanometer, a telephone, or some other alternating current detecting instrument. Balance in this instance is obtained by the dual process of altering the slide resistances together with the phase. When balance is obtained the magnitude of the P.D. is obtained, together with the angle of phase difference, cos ϕ and sin ϕ.

The instrument consists of two dials reading to 0·1 and 0·01 volt respectively, together with a slide wire for finer subdivisions. On the right side of the figure is seen the phase shifting transformer (supplied from the source of current used experimentally), together with a resistance and condenser, producing a pure rotating field. On the left is the alternating current galvanometer together with selector switch.

Readings can be relied upon to 0·1 per cent in magnitude and phase, and since there are two pointers at right angles on the dial of the phase shifting transformer, sin ϕ, cos ϕ, as well as ϕ itself, can be read off.

Consequently, the rectangular components of the vector quantity is obtained with great facility.

The method of adjusting scale is as follows :

(1) Calibrate wire or coils on D.C. against W, the standard cell—noting the current on A (dynamometer open scale type ammeter) when balance is

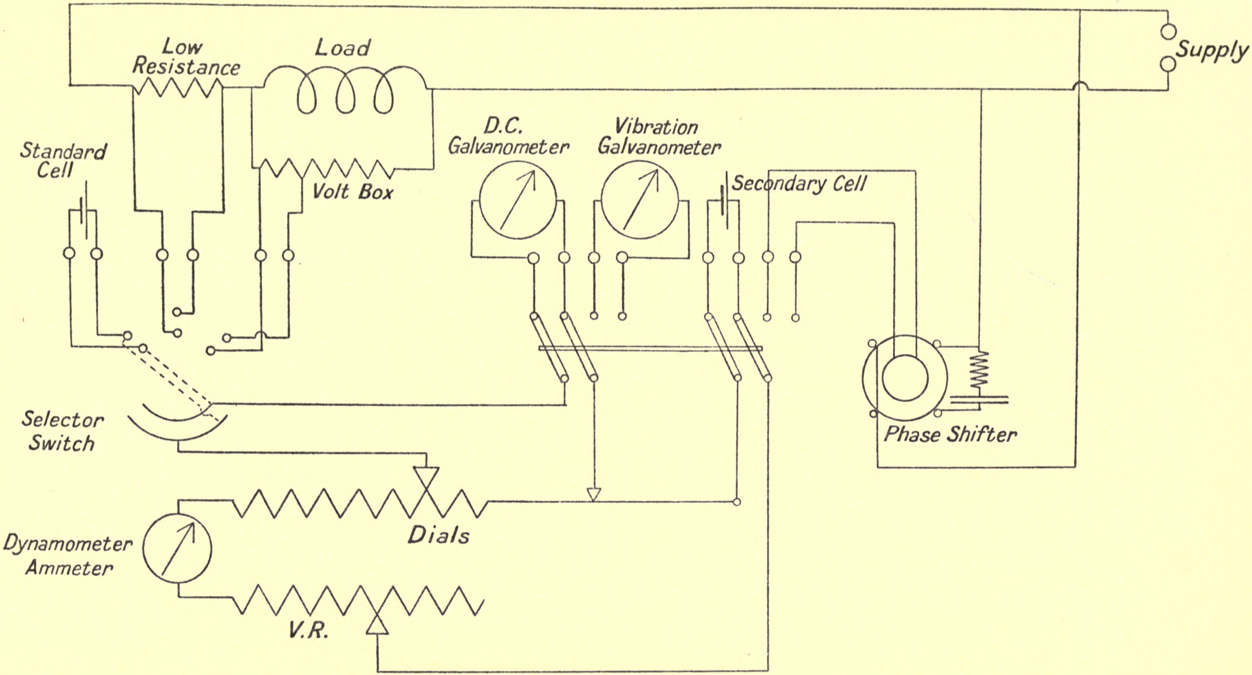


FIG. 157.—The A.C. Potentiometer.

obtained. With this current we obtain v volts per division.

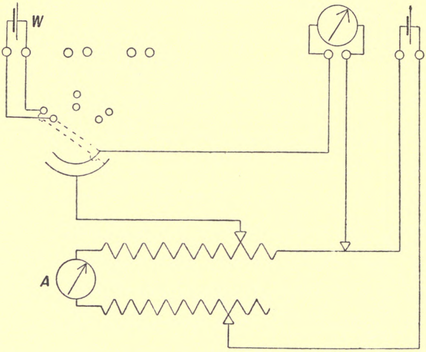


Fig. 158.—The A.C. Potentiometer ; calibration on continuous current.

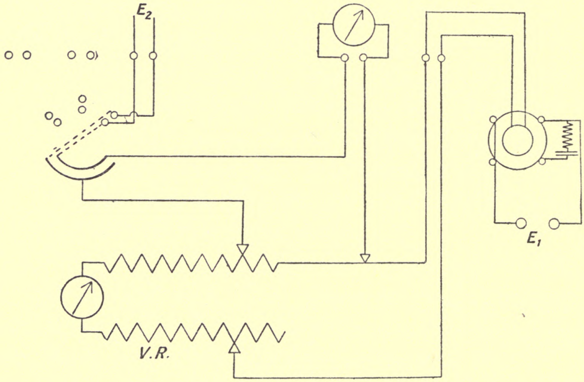


Fig. 159.—The A.C. Potentiometer ; measurement of an alternating voltage.

(2) Arrange as in Fig. 159, where E_1 is alternating current supply capable of having its phase shifted by phase shifter. E_2 is source of A.C. to be tested. Adjust

V.R. until the ammeter reads same as on direct current, shift phase until the maximum deflection is obtained with vibration galvanometer, shift the moving contact M until no deflection is obtained, then the volt drop per division is the same as in D.C. test.

By means of this instrument, current and potentials at high frequency can be measured, provided the wave is undamped, as in some systems of wireless telegraphy, suitable detectors being substituted for the galvanometer in this class of work.

For further information the reader should consult *Electrician*, August 1, 1913, article on "The Use of the Alternate Current Potentiometer for Measurements on Telegraph and Telephone Circuits," and also articles in the same paper, December 6 to January 10, 1908. Both articles by Dr. C. V. Drysdale.

The Deflection Potentiometer

These instruments are now used in the American Bureau of Standards for current and voltage measurements. In the ordinary method of using the potentiometer, the galvanometer merely indicates the direction of current or voltage, or zero.

In this case the galvanometer gives several figures of the result of test. The advantage appears to be that of rapidity combined with accuracy.

For a full discussion the reader must refer to the *Bulletin of the American Bureau of Standards*, vol. viii. No. 2, p. 395, "Deflection Potentiometers," by H. B. Brooke.

Consider Fig. 160.

Let AB be the potentiometer wire or resistance, then

$$r_1 + r_2 = \text{constant} = R_2, \text{ say,}$$

r_3 is a regulating resistance in series with the cell e_1 to adjust the volt drop in potentiometer.

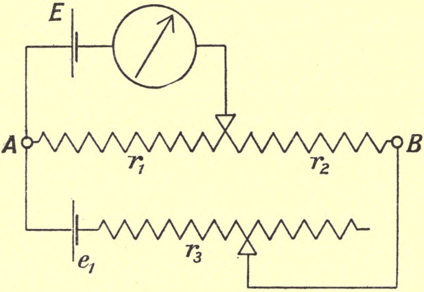


FIG. 160.—Principle of the deflection potentiometer.

Then it can be shown that if i is the galvanometer current, then

$$i = \frac{E - e_1 \dfrac{r_1}{r_1 + R_2}}{\dfrac{r_1 R_2}{r_1 + r_2} + r}.$$

Where r is the galvanometer resistance the term $e_1 \dfrac{r_1}{r_1 + R_2}$ is a definite drop per division usually measured by balancing against a standard cell.

Hence the numerator is the difference between the potentiometer reading and voltage E, and it is equal to

$$i\left(r + \frac{r_1 R_2}{r_1 + R_2}\right).$$

Consequently, if the term in brackets be constant, this difference can be read off directly in scale divisions.

If the connections are now arranged as shown below,

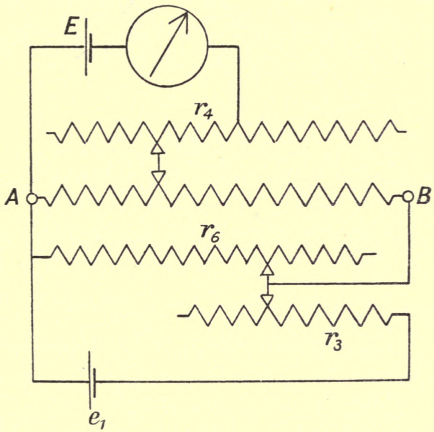


FIG. 161.—Connection of the deflection potentiometer.

AB is the potentiometer wire, or dials r_3, r_6, regulating resistance in series with the secondary cell e_1.

Let $r_6 + r_3$ in parallel $= r_7$ a constant. Then the resistance of the galvanometer circuit is clearly

$$r + r_4 + \left(\frac{r_1 r_8}{r_1 + r_8}\right),$$

where $r_8 = r_2 + r_7$ and r_4 is adjusted so that the sum of r_4 together with the term in brackets is kept constant. So that the total resistance in the galvanometer circuit is constant.

High Voltages.—The arrangement is as shown in Fig. 162. This merely adds an equivalent resistance of $R\frac{p-1}{p}$

and $\frac{R}{p}$ in parallel to the galvanometer circuit $= \frac{R}{p} \cdot \frac{p-1}{p}$.

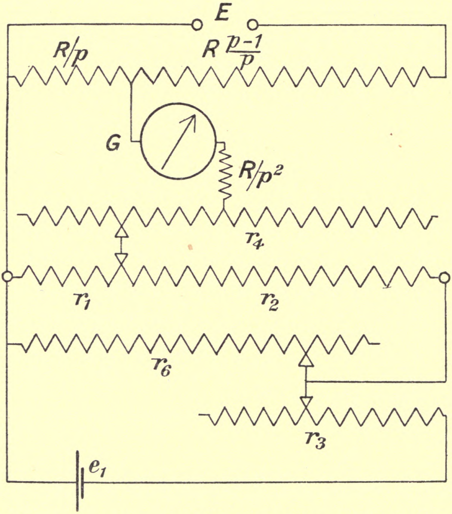


FIG. 162.—Measurement of high voltages by means of deflection potentiometer.

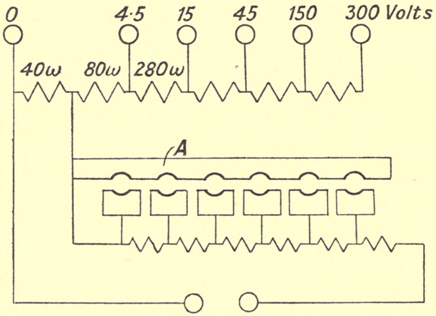


FIG. 163.—Connection of volt box.

To keep the total resistance of the volt box constant

when different ratios are employed, extra resistance $\left(=\frac{R}{p^2}\right)$ is put in series with the galvanometer. From Fig. 163 it will be seen that to measure up to 15 volts, we have 40 ohms and 360 ohms in parallel more than previously, hence the plug must be moved to A.

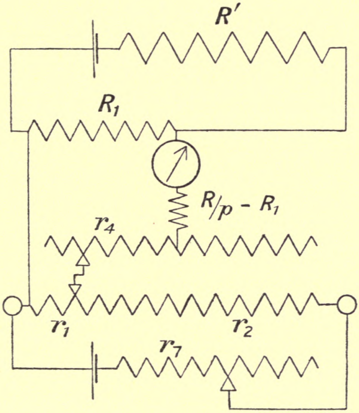


FIG. 164.—Measurement of current by means of deflection potentiometer.

For Low Voltages or Current Measurements, the arrangement is as shown in Fig. 164.

For measuring low voltages a resistance $\frac{R}{p}$ is put in series with the galvanometer and for current measurements a resistance $\left(\frac{R}{p}-R_1\right)$ is put in series with the shunt as shown in Fig. 164.

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THE END

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